SINGLE-INCLUSIVE JET PRODUCTION IN ELECTRON-NUCLEON COLLISIONS AT NNLO

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Introduction

- A large part of our knowledge about the internal structure of hadrons, and about QCD in general, comes from lepton-hadron scattering experiments.
- Traditionally we have studied the processes $lN \to lX$, $lN \to ljX$ and $lN \to lhX$
- Recently there has been a growing interest in $lN \to jX$ and $lN \to hX$ from both the theory and experimental communities. Applications include:
  - Measurement of the strong coupling constant
  - Extraction of fragmentation functions
  - Better our understanding of single-spin asymmetries in $pp^{\uparrow} \to hX$. Large all the way from fixed-target to collider energies
  - Improve our understanding of factorization:
    - Study of multiple parton interactions (MPI) and higher twist operators
    - Transverse-momentum-dependent (TMD) parton distributions

$$\boxed{\text{This talk: } lN \to jX \text{ through NNLO } (\mathcal{O}(\alpha^2\alpha_s^2)) \text{ in pQCD}}$$

Why NNLO? (I)

- (Semi)Inclusive DIS vs single-inclusive jet production

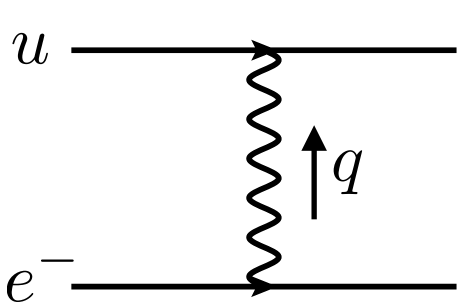
DIS $e^- N \to e^- X$

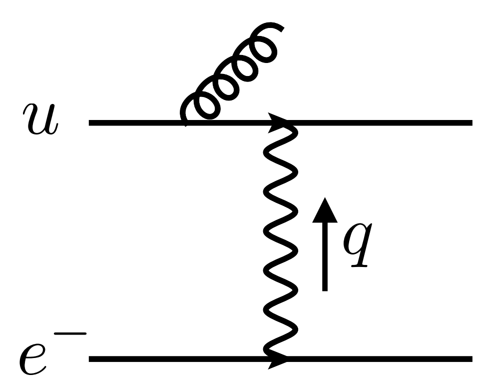
- Lepton observed
- Cut on $Q^2 = -q^2$
- Hard scale Q

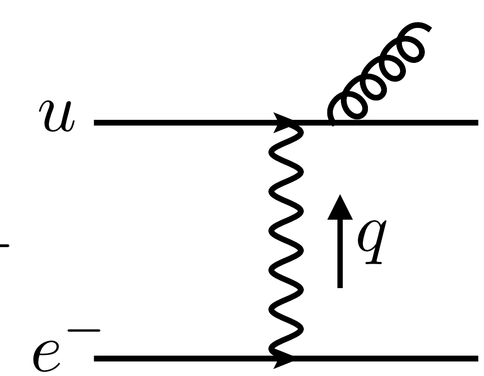
Inclusive jet production $e^- N \to j\, X$

- Inclusive over lepton
- Cut on p_T^{jet}
- Hard scale p_T^{jet}

- Equivalent at LO. Lepton recoils against jet
- At NLO, inclusive jet production probes the $Q^2 \sim 0$ region, unavailable in DIS
  - New singularities, new photon-initiated partonic channels (details later)
  - Large corrections from this region

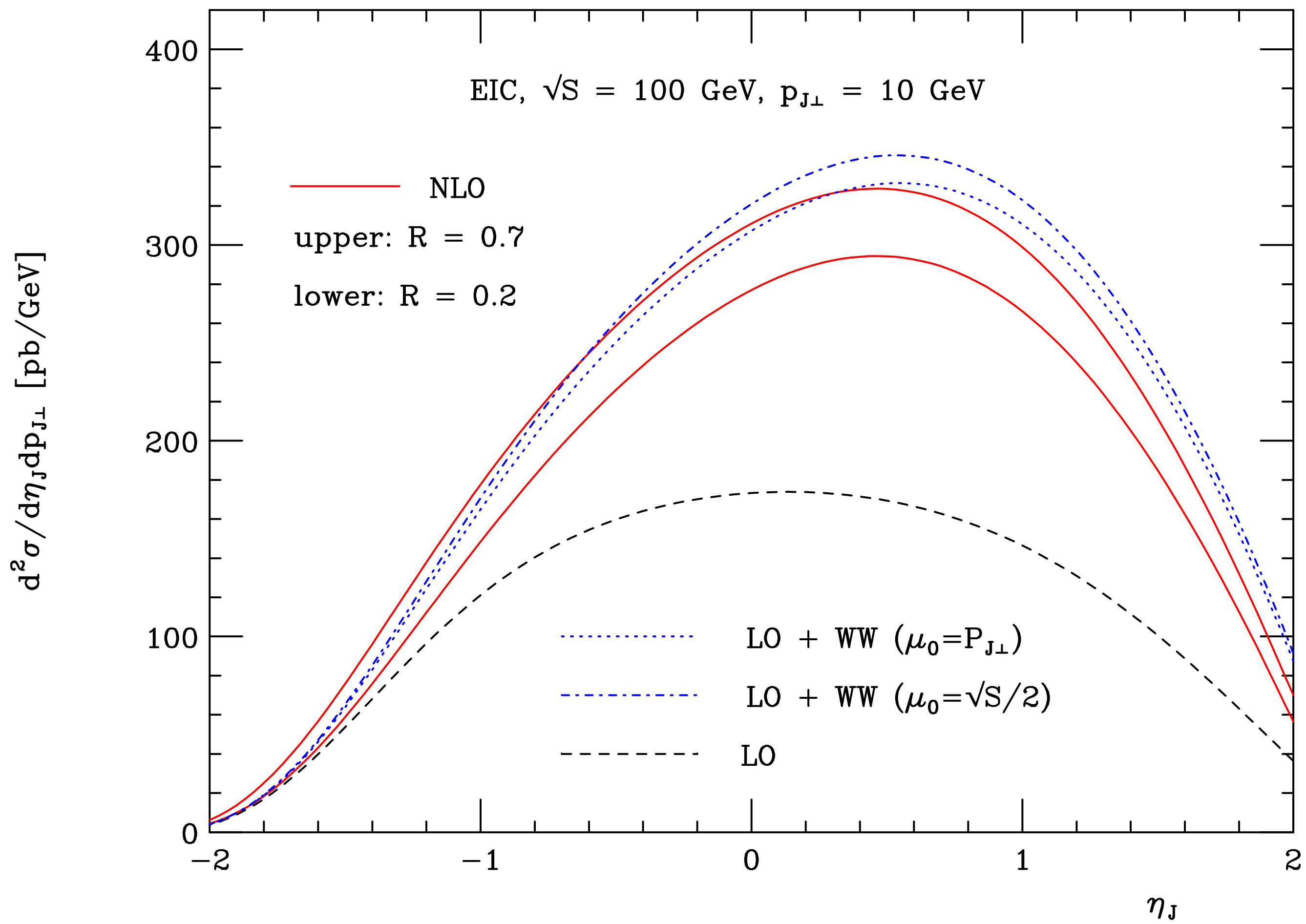
$\mathcal{M}_B =$ 

$\mathcal{M}_R =$ 

$+$ 

WHY NNLO? (II)

NLO QCD correction is small for inclusive DIS (~5%), but it is huge for single-inclusive jet production (>100%).



[Hinderer, Schlegel, Vogelsang '15]

NNLO needed for:

✤Precise theoretical predictions

✤Assessing stability of perturbative series

The Setup (LO)

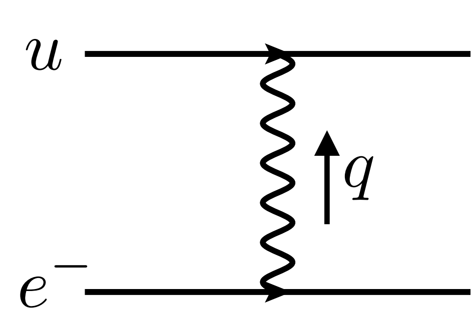
✤At leading order, the process $lN \to jX$ is trivial

$$d\sigma_{\mathrm{LO}} = \int \frac{d\xi_1}{\xi_1}\frac{d\xi_2}{\xi_2}\sum_q \left[f^1_{q/H} f^2_{l/l} d\hat{\sigma}^{(2,0)}_{ql} + f^1_{\bar{q}/H} f^2_{l/l} d\hat{\sigma}^{(2,0)}_{\bar{q}l} \right]$$

$$f^k_{i/j} = f_{i/j}(\xi_k) \qquad f_{l/l}(\xi) = \delta(1-\xi) \qquad d\hat{\sigma}^{(m,n)}_{ql} \propto \alpha^n \alpha_s^m$$

✤ The partonic cross sections are

$$d\hat{\sigma}^{(2,0)}_{ql} = \frac{(4\pi\alpha)^2}{8s}\, e_q^2\, d\Phi_B(p_3, p_4; p_1, p_2) |\mathcal{M}_B|^2 J^{(1)}(p_3)$$

$\mathcal{M}_B =$ 

The Setup (NLO)

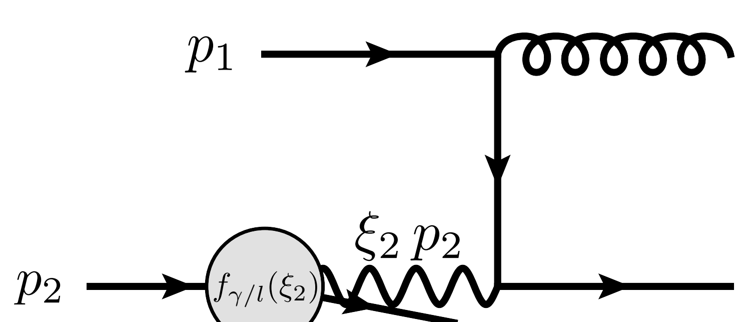
- At NLO
  - Real and virtual corrections to ql channel.
  - gl channel opens up

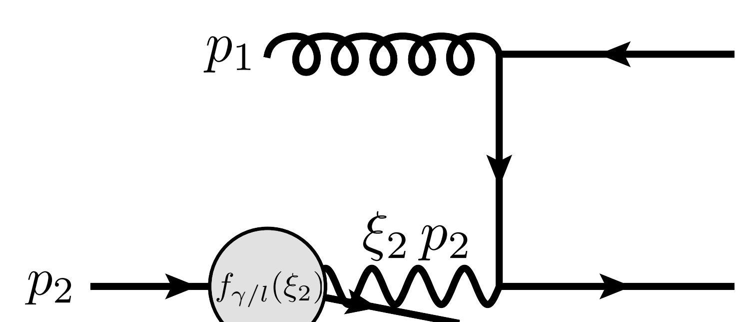
$$
\mathrm{d}\hat{\sigma}_{ql}^{(2,1)} = \int \mathrm{d}\Phi_R |\mathcal{M}_R^{(ql)}|^2 J^{(2)}(p_3,p_5) + \int \mathrm{d}\Phi_B |\mathcal{M}_V^{(ql)}|^2 J^{(1)}(p_3)
$$

$$
\mathrm{d}\hat{\sigma}_{gl}^{(2,1)} = \int \mathrm{d}\Phi_R |\mathcal{M}_R^{(gl)}|^2 J^{(2)}(p_3,p_5)
$$

- We handle the QCD soft and collinear IR singularities with NLO antenna subtraction, and mass factorization as usual
- We also handle the QED singularity $(p_l||p_{l'})$ with NLO antenna subtraction and mass factorization

$\Longrightarrow$ Introduce (LO) photon-initiated processes





The Setup (NLO)

✤NLO correction to $lN \to jX$:

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int \frac{\mathrm{d}\xi_1}{\xi_1}\frac{\mathrm{d}\xi_2}{\xi_2}\Bigg\{ f^1_{g/H} f^2_{l/l} \mathrm{d}\hat{\sigma}^{(2,1)}_{gl} + f^1_{g/H} f^2_{\gamma/l} \mathrm{d}\hat{\sigma}^{(1,1)}_{g\gamma}$$

$$+ \sum_q \Bigg[f^1_{q/H} f^2_{l/l} \mathrm{d}\hat{\sigma}^{(2,1)}_{ql} + f^1_{\bar{q}/H} f^2_{l/l} \mathrm{d}\hat{\sigma}^{(2,1)}_{\bar{q}l}$$

$$+ f^1_{q/H} f^2_{\gamma/l} \mathrm{d}\hat{\sigma}^{(1,1)}_{q\gamma} + f^1_{\bar{q}/H} f^2_{\gamma/l} \mathrm{d}\hat{\sigma}^{(2,1)}_{\bar{q}\gamma} \Bigg] \Bigg\}$$

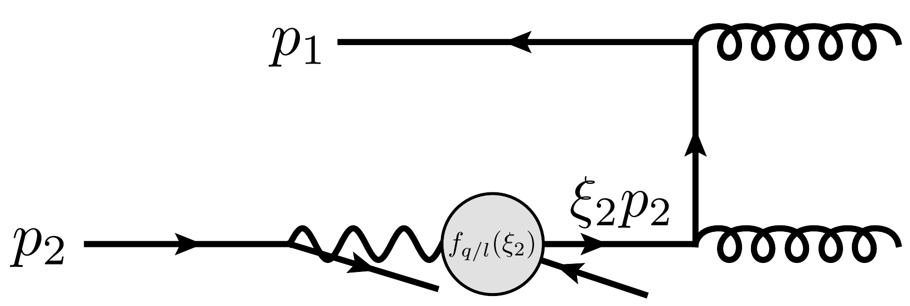
✤Perturbative photon-in-lepton distribution (Weizsäcker-Williams)

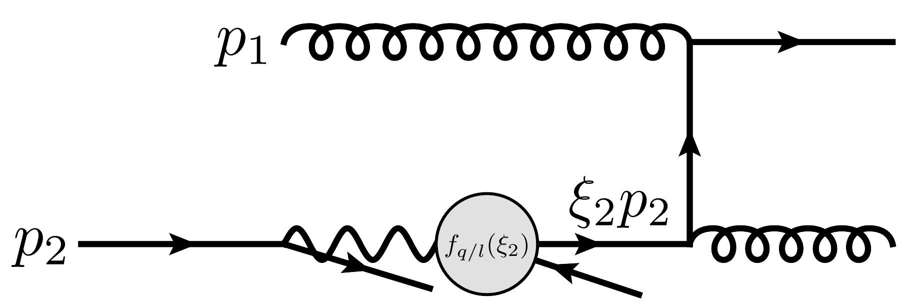
$$f_{\gamma/l}(\xi) = \frac{\alpha}{2\pi} P_{\gamma l}(\xi) \left[\ln\left(\frac{\mu^2}{\xi^2 m_l^2} \right) - 1 \right] + \mathcal{O}(\alpha^2)$$

$$P_{\gamma l}(\xi) = \frac{1 + (1-\xi)^2}{\xi}$$

THE SETUP (NNLO)

- At NNLO
  - Genuine NNLO corrections to ql and gl channels
    - QED IR divergencies ($p_l||p_{l'}$, $p_l||p_{l'}||p_q$) handled with antenna subtraction
      [Daleo, Gehrmann, Gehrmann-De Ridder, Luisoni '10;
      Boughezal, Gehrmann-De Ridder, Ritzmann '10;
      Gehrmann, Gehrmann-De Ridder, Ritzmann '12]
    - QCD IR divergencies handled with N-Jettiness subtraction
      [Boughezal, Focke, Liu, Petriello '15;
      Gaunt, Stahlhofen, Tackmann, Walsh '15]
  - NLO corrections to $q\gamma$ and $g\gamma$ channels. All singularities treated with antennae.
  - New $q\bar{q}$ and qg channels





The Setup (NNLO)

- Quark-in-lepton distribution computed perturbatively from DGLAP equation

$$\mu^2 \frac{\partial f_{q/l}}{\partial \mu^2}(\xi,\mu^2) = e_q^2 \frac{\alpha}{2\pi} \int_\xi^1 \frac{\mathrm{d}z}{z} \left[P_{q\gamma}^{(0)}(z)\, f_{\gamma/l}\left(\frac{\xi}{z},\mu^2\right) + \frac{\alpha}{2\pi} P_{ql}^{(1)}(z)\, f_{l/l}\left(\frac{\xi}{z},\mu^2\right)\right]$$

$$P_{q\gamma}^{(0)}(x) = x^2 + (1-x)^2$$

$$P_{ql}^{(1)}(x) = -2 + \frac{20}{9x} + 6x - \frac{56x^2}{9} + \left(1 + 5x + \frac{8x^2}{3}\right)\log(x) - (1+x)\log^2(x)$$

- Boundary condition $f_{q/l}(\xi, m_l^2) = 0$

$$f_{q/l}(\xi,\mu^2) = e_q^2 \left(\frac{\alpha}{2\pi}\right)^2 \left\{ \left[\frac{1}{2} + \frac{2}{3\xi} - \frac{\xi}{2} - \frac{2\xi^2}{3} + (1+\xi)\log\xi\right] \log^2\left(\frac{\mu^2}{m_l^2}\right) \right.$$
$$\left. + \left[-3 - \frac{2}{\xi} + 7\xi - 2\xi^2 + \left(-5 - \frac{8}{3\xi} + \xi + \frac{8\xi^2}{3}\right)\log\xi - 3(1+\xi)\log^2\xi \right] \log\left(\frac{\mu^2}{m_l^2}\right) \right\}$$

1-JETTINESS SUBTRACTION

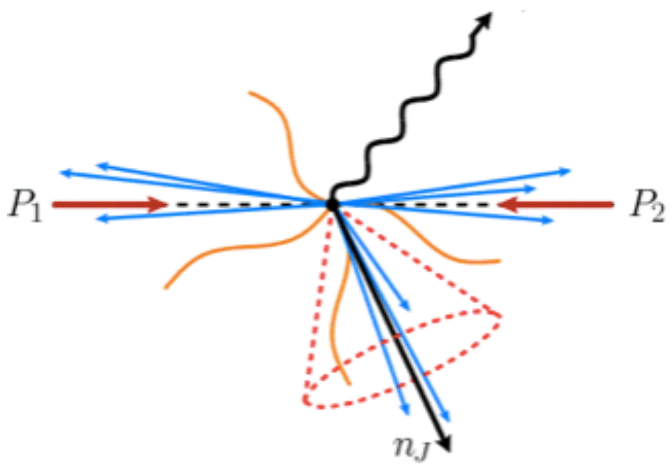
✤ Starting point: dimensionless 1-jettiness event shape [Kang, Lee, Stewart '13]

$$\mathcal{T}_1 = \frac{2}{Q^2} \sum_i \min\{p_B \cdot q_i, p_J \cdot q_i\}$$

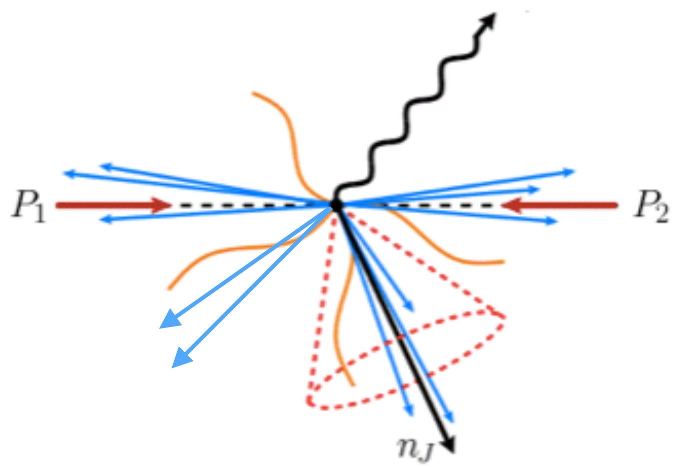
✤ p_B: beam axis
✤ p_J : leading jet axis
✤ q_i : outgoing parton momenta

1 jet $\xleftarrow{\text{Small}}$ $\mathcal{T}_1$ $\xrightarrow{\text{Large}}$ At least 2 jets

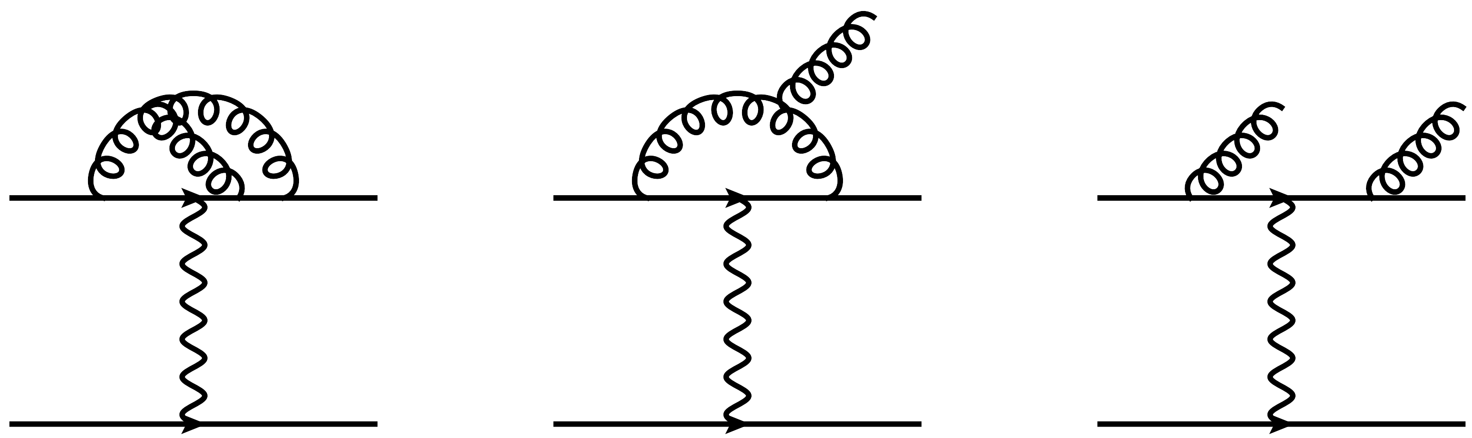
Contributions from
✤ Two-loop
✤ Soft and collinear radiation



Contributions with at least 2 hard jets
✤ NLO two-jet calculation. Use known results/tools



1-Jettiness Subtraction



- Introduce internal cutoff $\mathcal{T}_1^{cut}$. Partition RR and RV phase space

$$
\begin{aligned}
\mathrm{d}\sigma_{ql}^{(2,2)} &= \int \mathrm{d}\Phi_{\mathrm{B}}\,|\mathcal{M}_{\mathrm{VV}}|^2 + \int \mathrm{d}\Phi_{\mathrm{R}}\,|\mathcal{M}_{\mathrm{RV}}|^2\,\theta_1^{<} + \int \mathrm{d}\Phi_{\mathrm{RR}}\,|\mathcal{M}_{\mathrm{RR}}|^2\,\theta_1^{<} \\
&\quad + \int \mathrm{d}\Phi_{\mathrm{R}}\,|\mathcal{M}_{\mathrm{RV}}|^2\,\theta_1^{>} + \int \mathrm{d}\Phi_{\mathrm{RR}}\,|\mathcal{M}_{\mathrm{RR}}|^2\,\theta_1^{>} \\
&\equiv \mathrm{d}\sigma_{ql}^{(2,2)}(\mathcal{T}_1 < \mathcal{T}_1^{cut}) + \mathrm{d}\sigma_{ql}^{(2,2)}(\mathcal{T}_1 > \mathcal{T}_1^{cut})
\end{aligned}
$$

$$
\theta_1^{<} = \theta(\mathcal{T}_1^{cut} - \mathcal{T}_1) \qquad \theta_1^{>} = \theta(\mathcal{T}_1 - \mathcal{T}_1^{cut})
$$

NNLO Cross Section Below 1-Jettiness Cut

All-orders resummation of $\mathcal{T}_1$ in DIS for the limit $\mathcal{T}_1 \ll 1$ known [Kang, Lee, Stewart '13]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_1} = \int \mathrm{d}\Phi_B \int \mathrm{d}t_J \mathrm{d}t_B \mathrm{d}k_S \, \delta\left(\mathcal{T}_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times \sum_q J_q(t_J,\mu)\, S(k_S,\mu) H_q(\Phi_2,\mu) B_q(t_B,x,\mu) + \ldots$$

Power corrections $\propto \mathcal{T}_1^{cut}$
Small for small cutoffs

- Expand through $\mathcal{O}\left(\alpha^2\alpha_s^2\right)$ to obtain $\mathrm{d}\sigma_{ql}^{(2,2)}(\mathcal{T}_1 < \mathcal{T}_1^{cut})$, $\mathrm{d}\sigma_{gl}^{(2,2)}(\mathcal{T}_1 < \mathcal{T}_1^{cut})$
- All pieces known to this order
  - Jet function $J_q(t_J,\mu)$ [Becher and Neubert '06; Becher and Bell '10]
  - Beam function $B_q(t_B,x,\mu)$ [Gaunt, Stahlhofen, Tackmann '14]
  - Soft function $S(k_S,\mu)$ [Boughezal, Liu, Petriello '15]
  - Hard function $H_q(\Phi_2,\mu)$ [Matsuura, van der Marck, van Neerven '88 ; Becher, Neubert, Pecjak '06]

DISTRESS

DISTRESS: DIS Through a Robust Enabling Subtraction Scheme

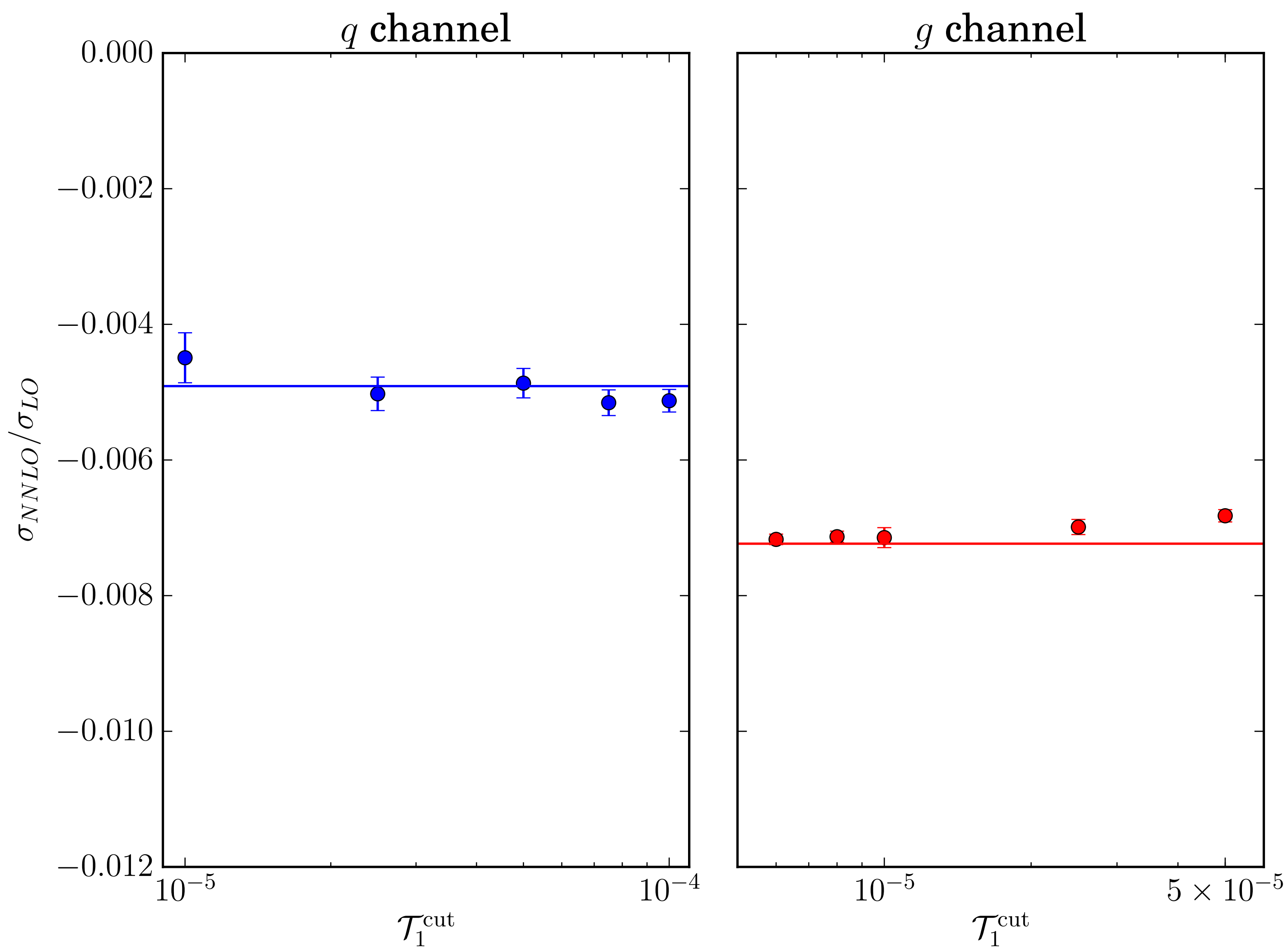
Soon to become publicly available

- ✤Parton-level event generator for inclusive jet production in eN collisions at NNLO
- ✤Fully differential
- ✤Arbitrary cuts on jet and final state lepton
- ✤Parallelized Monte Carlo integration

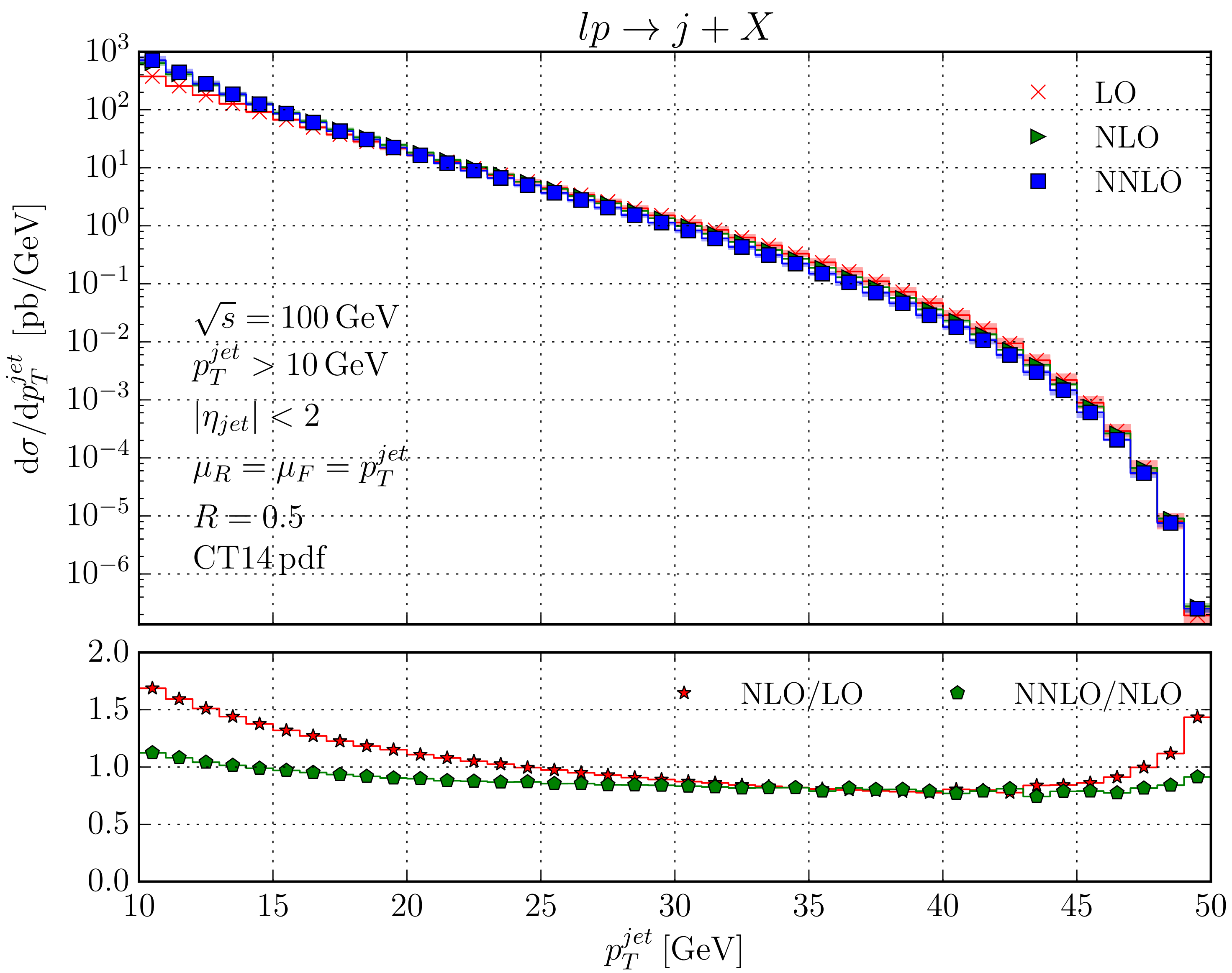
Results: Validation

Inclusive DIS cross section with $\sqrt{s} = 100\,\text{GeV},\ Q^2 > 100\,\text{GeV}^2,\ \mu_R = \mu_F = Q$ and CT14nnlo PDFs

- Check cutoff independence of result
- Determine range of cutoffs for which power corrections are negligibly small
- Agreement with NNLO inclusive cross section computed with structure functions [Zijlstra, van Neerven '92]



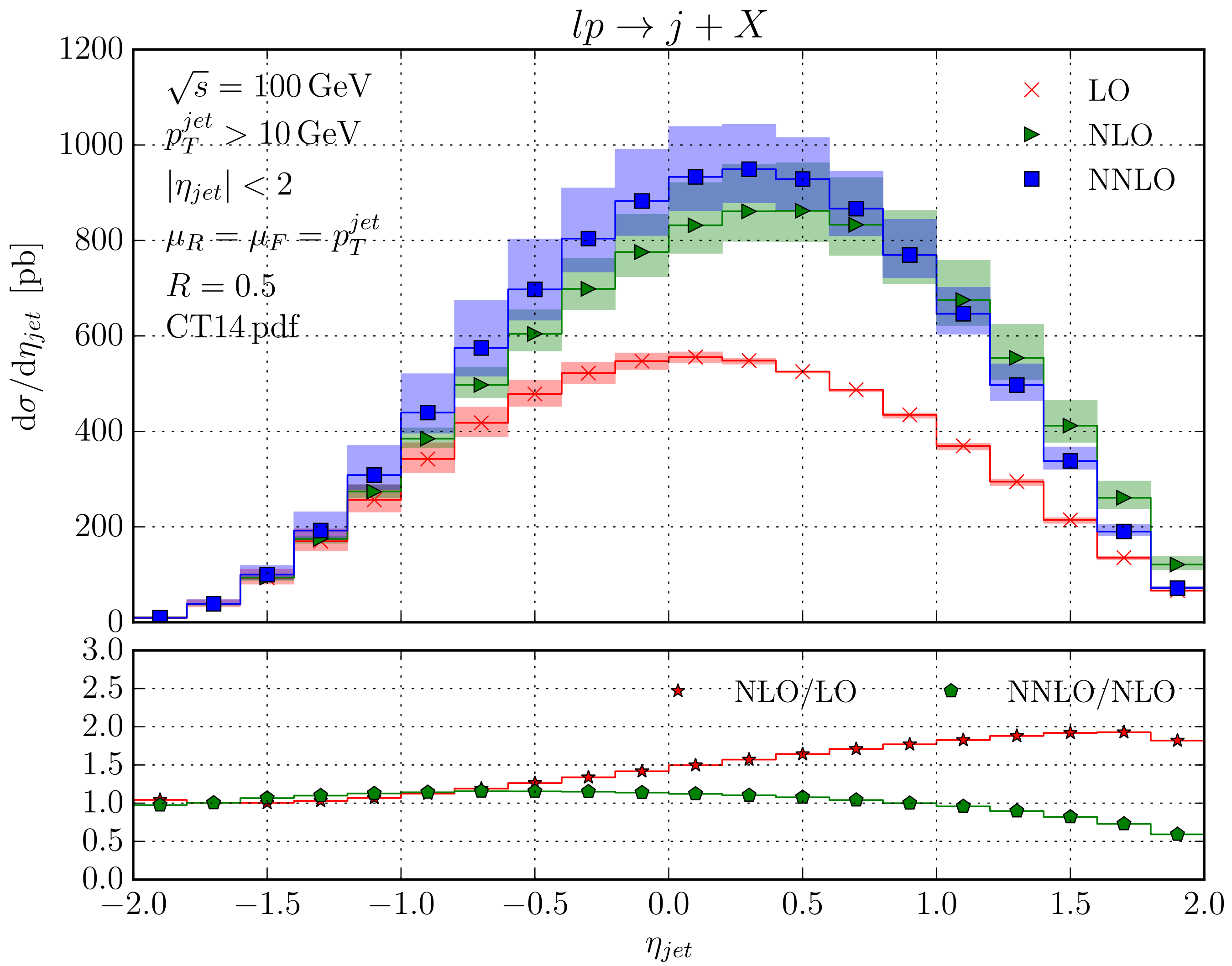
RESULTS: JET TRANSVERSE MOMENTUM



✤NNLO correction is small. Changes NLO result by no more than 10%

✤Shift is positive for low p_T^{jet}, and negative at the end of the spectrum

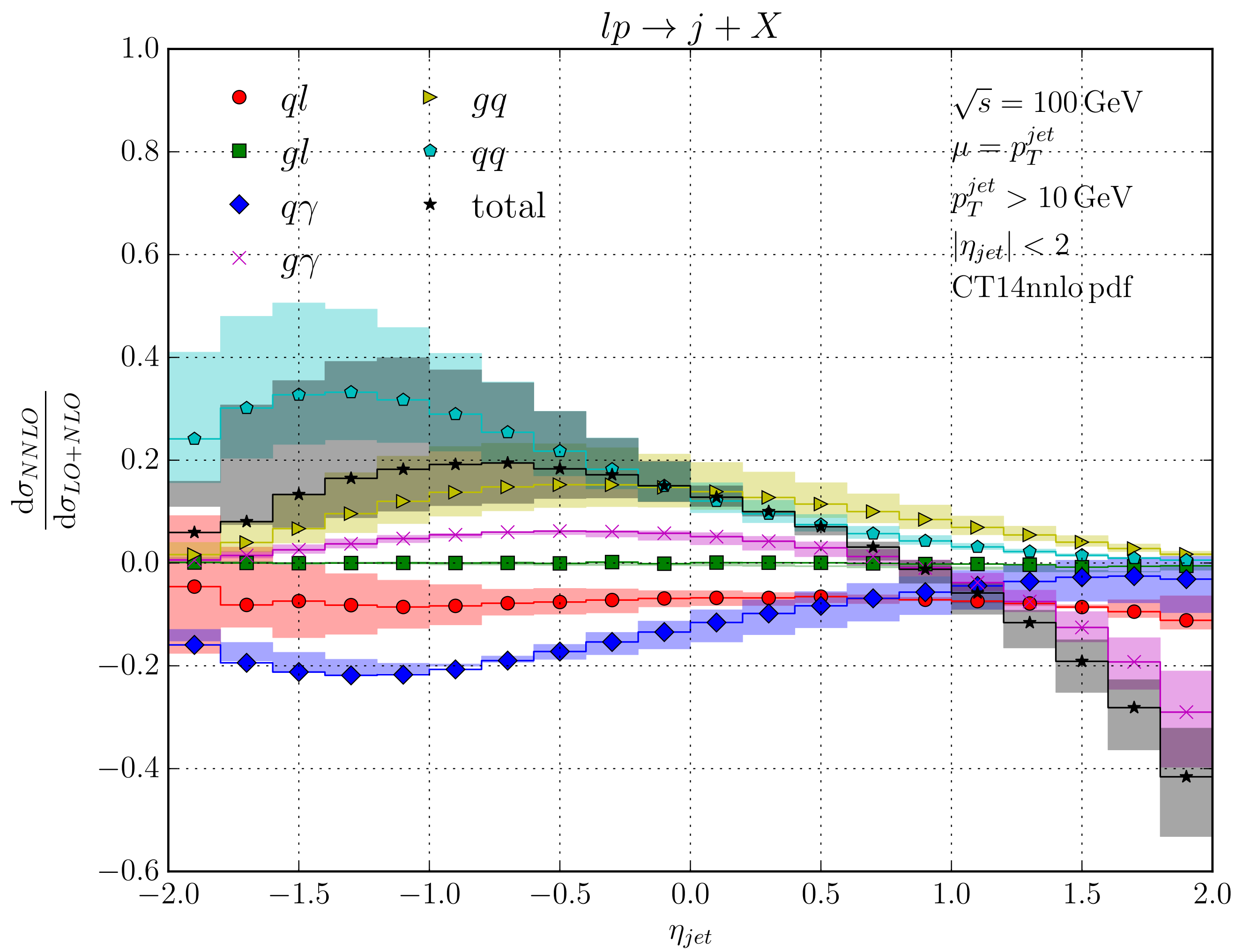
RESULTS: JET RAPIDITY (I)



✤NNLO correction is small for $\eta_{jet} < 1$, sizable as $\eta_{jet} \to 2$

✤NNLO scale uncertainty in the region $\eta_{jet} < 0$ larger that at NLO

RESULTS: JET RAPIDITY (II)



- ✤Large scale uncertainty in the region $\eta_{jet} < 0$ is driven by the quark-quark channel, which is effectively LO at $\mathcal{O}(\alpha^2\alpha_s^2)$
- ✤As $\eta_{jet} \to 2$ the NNLO correction is largely dominated by the gluon-photon channel
- ✤No single partonic channel furnishes a good approximation to the shape of the full NNLO correction

Summary and Outlook

- We have performed a full calculation of the $\mathcal{O}(\alpha^2\alpha_s^2)$ perturbative corrections to jet production in electron-nucleon collisions, using N-jettiness subtraction
- We have shown that upon integration over the final-state hadronic phase we reproduce the known NNLO result for the inclusive structure functions
- We have implemented our results in a fully differential parton-level event generator DISTRESS
- We have shown numerical results for jet production at a proposed future EIC
  - Several new partonic channels appear at the $\mathcal{O}(\alpha^2\alpha_s^2)$ level, which have an important effect on the kinematic distributions of the jet
  - No single partonic channel furnishes a good approximation to the full NNLO result
  - The magnitudes of the corrections we find indicate that higher-order predictions will be an important part of achieving the precision understanding of proton structure desired at the EIC

- Soon(ish):
  - Extend calculation to single-inclusive hadron production in eN collisions
  - Make DISTRESS publicly available

THANK YOU!