Transverse-momentum resummation for top-quark pair production at hadron colliders

Hayk Sargsyan

in collaboration with R. Bonciani, S. Catani, M. Grazzini

University of Zurich

HP2.6, Buenos Aires,
September 8, 2016

Outline

- Motivation
- q_T-resummation
    - Preliminary results
- q_T-subtraction
    - Results
- Summary

Top quark

Mass of the top quark obtained through combining the measurements at the Tevatron and LHC colliders is $m_t = 173.34 \pm 0.27\,(stat)\ \pm 0.71\,(syst)\,\mathrm{GeV}$
[ATLAS and CDF and CMS and D0 Collaborations (2014)].

- Strong coupling to the Higgs boson
- Crucial to the hierarchy problem

Top quark pair production

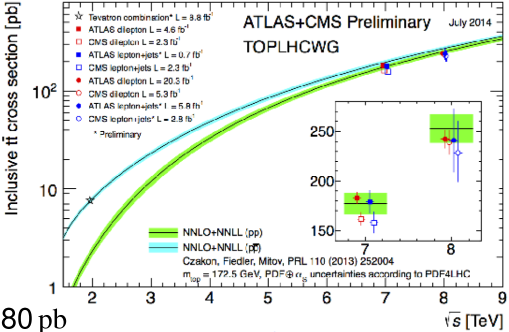
- The top quark pair production is the main source of the top quark events in the Standard Model (SM).
- Many New Physics models involve heavy top partners which then decay into a top quark pair.

The study of the $t\bar{t}$ pair production at hadron colliders can

- shed light on the electroweak symmetry breaking mechanism.
- provide information on the backgrounds of many NP models.

Top quark pair production

- Because of its large mass the top quark decay before hadronization, allowing for a better experimental measurements.



- $\sigma_{t\bar{t}}(14\,\mathrm{TeV}) \sim 980\,\mathrm{pb}$
  $\mathcal{L} \sim 10^{32}\,\mathrm{cm}^2\mathrm{s}^{-1}$ $\longrightarrow$ 1 event/10 s

More precise calculations are needed from the theory side

QCD corrections

- NLO QCD corrections are calculated by [Nason, Dawson and Ellis (1988), Beenakker, Kuijf, van Neerven and Smith (1989), Beenakker, van Neerven, Meng, Schuler and Smith (1989)].
- NNLO corrections in the threshold region are worked out in [Ahrens, Ferroglia, Neubert, Pecjak, Yang (2010), Beneke, Falgari, Schwinn (2010), Cacciari, Czakon, Mangano, Mitov, Nason (2012)].
- The calculation of the full NNLO QCD corrections was completed for the total cross section and for the $t\bar{t}$ asymmetry. [Barnreuther, Czakon, Mitov (2012), Czakon, Mitov (2012, 2013), Czakon, Fiedler, Mitov (2013, 2014)].
- The first NNLO results for differential distributions at the Tevatron and LHC have been computed [Czakon, Heymes, Mitov (2016), Czakon, Fiedler, Heymes, Mitov (2016)].
- Other computations of differential distributions are underway [Abelof, Gehrmann-De Ridder, Maierhofer (2014), Abelof, Gerhrmann-de Ridder (2014), Abelof, Gerhrmann-de Ridder, Majer (2016)].
- Last year the NNLO corrections for all off-diagonal partonic channels have been computed by our group [Bonciani, Catani, Grazzini, HS, Torre].

q_T distribution

- When $q_T^2 \sim M^2$, $\alpha_S(M^2)$ is small, and the standard fixed order expansion is theoretically justified.
- When $q_T^2 \ll M^2$ large logarithms of the form $\alpha_S^n \log(M^2/q_T^2)$ appear, due to soft and collinear gluon emissions. Effective expansion variable is the $\alpha_S^n \log(M^2/q_T^2)$, which can be ~ 1 even for small α_S. These large logarithms need to be resummed to all orders in α_S, in order to get reliable predictions over the whole range of the transverse momenta.

The resummation of large logs results in exponentiating these large logarithmic terms

$$\sigma^{(res)} \sim \sigma^{(0)} \underset{\text{hard-virtual}}{C(\alpha_S)} exp \left\{ \underset{\text{LL}}{L g_1(\alpha_S L)} + \underset{\text{NLL}}{g_2(\alpha_S L)} + \underset{\text{NNLL}}{\alpha_S g_3(\alpha_S L)} + \ldots \right\} .$$

Resummation for the $t\bar{t}$ production

Production of coloured particles imposes additional complications compared to the production of a colourless system.

- Soft and collinear QCD radiation from the final state particles
- Colour flow between initial and final state particles leading to non-trivial colour correlations

The top quark is massive

- The collinear limit is not singular ⟶ LL structure unaffected
- Additional NLL from large-angle soft radiation

Resummation for the $t\bar{t}$ production

- The first attempt to develop a q_T-resummation formalism at next-to-leading logarithmic (NLL) accuracy for $t\bar{t}$ production was done in [Berger, Meng (1994), Mrenna, Yuan (1997)]. However, they did not consider colour mixing between singlet and octet final states and missed the initial-final gluon exchange.
- The resummation for the $t\bar{t}$ q_T spectrum, based on soft collinear effective theory (SCET), was performed at NNLL+NLO. [Zhu, Li, Li, Shao, Yang (2013)]. This work is limited to the study of the q_T cross section after integration over the azimuthal angles of the produced heavy quarks.
- The q_T-resummation in QCD was performed at the fully-differential level with respect to the kinematics of the produced heavy quarks. [Catani, Grazzini, Torre (2014)].

The resummation procedure at small q_T

$h_1(P_1) + h_2(P_2) \to Q(p_3) + \bar{Q}(p_4) + X.$

- Consider the most general fully-differential cross section

$$\frac{d\sigma(P_1, P_2; \mathbf{q_T}, M, y, \Omega)}{d^2\mathbf{q_T} dM^2 dy d\Omega} ,$$

  where P_1 and P_2 are the momenta of incoming hadrons, $\mathbf{q_T}$, M and y are the transverse momentum vector, invariant mass and rapidity of the $Q\bar{Q}$ pair, Ω is a set of two additional independent kinematical variables that specify the angular distribution of heavy quarks with respect to the momentum q of the $Q\bar{Q}$ pair. For instance $\Omega = \{y_3, \phi_3\}$.
- Decompose the cross section in a singular and a regular part

$$d\sigma = d\sigma^{(\mathrm{sing})} + d\sigma^{(\mathrm{reg})} .$$

- $d\sigma^{(\mathrm{sing})}$ embodies all the singular terms in the limit $q_T \to 0$.
- $d\sigma^{(\mathrm{reg})}$ includes the remaining non-singular terms.

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should be replaced by $d\sigma^{res}$

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The all-order resummation formula

- Is obtained by working in impact parameter **b** space.

$$\frac{d\sigma^{(\mathrm{res})}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\Omega} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{cc}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\mathbf{q_T}}\,S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[(\boldsymbol{H\Delta})C_1\,C_2\right]_{c\bar{c};a_1a_2}\times$$

$$f_{a_1/h_1}(x_1/z_1,\,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,\,b_0^2/b^2)\,.$$

$b_0 = 2e^{-\gamma_E}$ (γ_E is the Euler number).

$$x_1 = \frac{M}{\sqrt{s}}e^{+y} \qquad x_2 = \frac{M}{\sqrt{s}}e^{-y}\,.$$

$$\ln S_c(M,b) = \int_{M^2}^{b_0^2/b^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\ln\frac{M^2}{q^2} + B_c(\alpha_S(q^2))\right]\,.$$

LL: $A_c^{(1)}$,   NLL: $A_c^{(2)}, B_c^{(1)}$.

The all-order resummation formula

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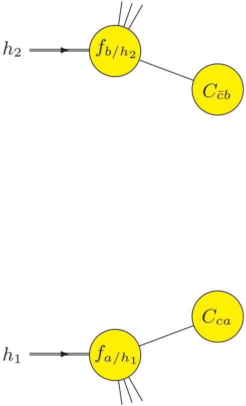

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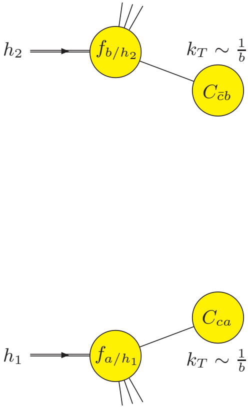

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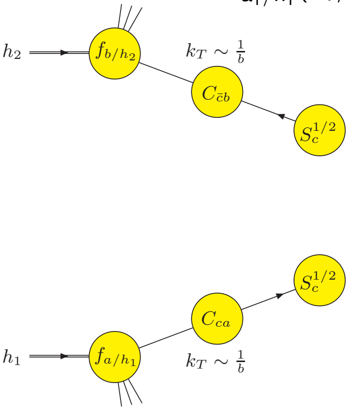

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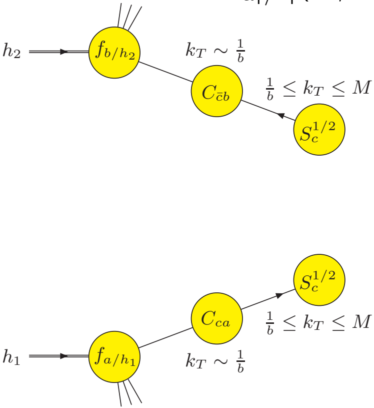

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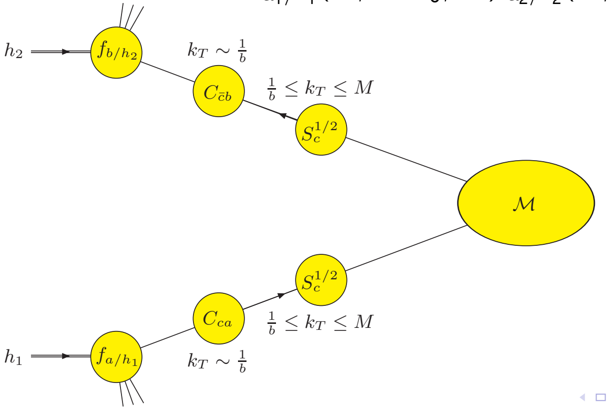

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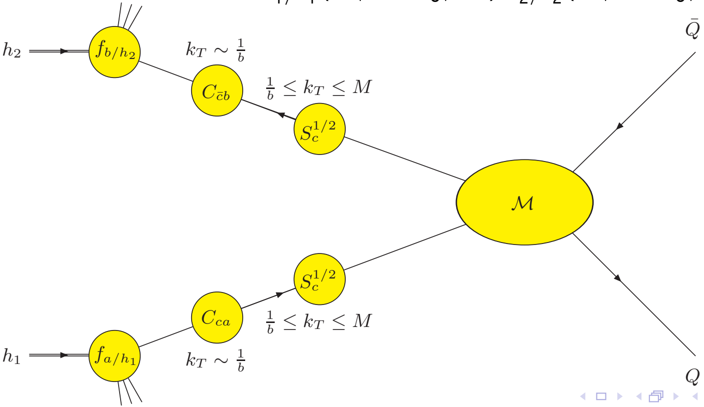

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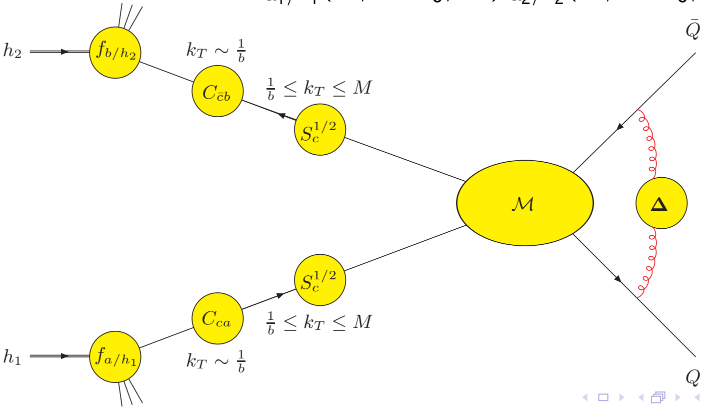

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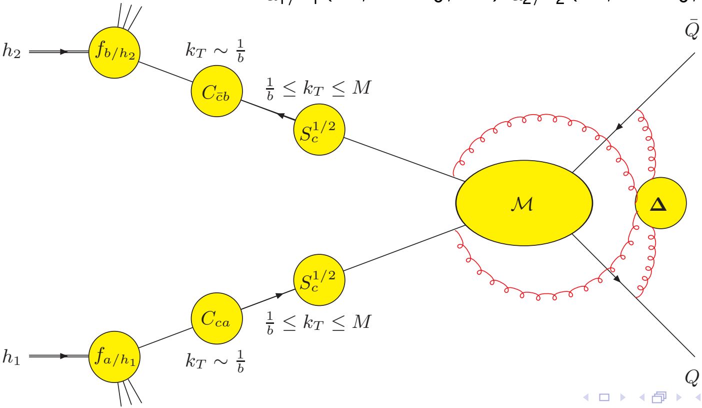

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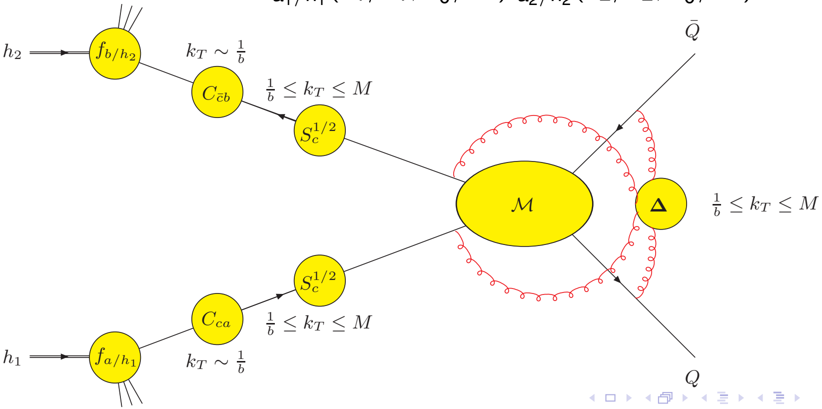


The soft-parton factor $\boldsymbol{\Delta}$

$$[(\boldsymbol{H\Delta})C_1C_2]_{q\bar{q};a_1a_2} = \boldsymbol{H\Delta}_{q\bar{q}}C_{qa_1}\left(\alpha_S\left(b_0^2/b^2\right)\right)C_{\bar{q}a_2}\left(\alpha_S\left(b_0^2/b^2\right)\right) .$$

$$[(\boldsymbol{H\Delta})C_1C_2]_{gg;a_1a_2} = \boldsymbol{H\Delta}_{gg}C_{ga_1}^{\mu_1\nu_1}\left(\mathbf{b};\alpha_S\left(b_0^2/b^2\right)\right)\cdot C_{ga_2}^{\mu_2\nu_2}\left(\mathbf{b};\alpha_S\left(b_0^2/b^2\right)\right) .$$

$$\boldsymbol{H\Delta}_{q\bar{q}} = \frac{\langle\tilde{\mathcal{M}}_{q\bar{q}\to Q\bar{Q}}|\boldsymbol{\Delta}|\tilde{\mathcal{M}}_{q\bar{q}\to Q\bar{Q}}\rangle}{\alpha_S^2(M^2)\left|\mathcal{M}^{(0)}_{q\bar{q}\to Q\bar{Q}}(p_1,p_2,p_3,p_4)\right|^2} .$$

$$|\tilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}\rangle = \left[1-\tilde{I}_{c\bar{c}\to Q\bar{Q}}\right]|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle .$$

$$\boldsymbol{\Delta}(b,M;y_{34},\phi_3) = \mathbf{V}^\dagger(b,M;y_{34})\mathbf{D}(\alpha_S\left(b_0^2/b^2\right);\phi_{3b},y_{34})\mathbf{V}(b,M;y_{34}) .$$

$$\mathbf{V}(b,M;y_{34}) = \bar{P}_q\mathrm{exp}\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\boldsymbol{\Gamma}_t(\alpha_S(q^2);y34)\right\} .$$

- $\boldsymbol{\Gamma}_t$ is the soft anomalous dimension matrix.
- **D** is the azimuthal-correlation matrix.
  $\langle\mathbf{D}(\alpha_S;\phi_{3b},y_{34})\rangle_{\mathrm{av.}} = 1$.
- All the perturbative coefficients are computed at NLO QCD.

The soft anomalous dimension operator

$$\boldsymbol{\Gamma}_t(\alpha_S; y_{34}) = \frac{\alpha_S}{\pi}\boldsymbol{\Gamma}_t^{(1)}(y_{34}) + \left(\frac{\alpha_S}{\pi}\right)^2 \boldsymbol{\Gamma}_t^{(2)}(y_{34}) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \boldsymbol{\Gamma}_t^{(n)}(y_{34})\,.$$

- The colour basis: *s*-channel singlet-octet exchange tensors
  - $c_1^{q\bar{q}} = \delta_{ij}\delta_{kl}\,, \quad c_2^{q\bar{q}} = t^c_{ji}t^c_{kl},$
  - $c_1^{gg} = \delta_{ab}\delta_{kl}\,, \quad c_2^{gg} = if^{abc}t^c_{kl}\,, \quad c_3^{gg} = d^{abc}t^c_{kl}.$

$$\Gamma_{ij} = \frac{1}{\langle c_i|c_i\rangle}\,\langle c_i|\,\boldsymbol{\Gamma}_t\,|c_j\rangle\ .$$

The soft anomalous dimension matrix is non-diagonal
→ needs to be diagonalized!

Transform the original basis $|I\rangle = R_{cI}\,|c\rangle$ by the diagonalization matrix

$$R^{-1}\,\Gamma\,R = \Gamma^{\rm diag}\,.$$

In the new basis the matrix element of soft anomalous dimension operator is given by

$$\Gamma^{\rm diag}_{IJ} = \langle\mathcal{X}_I|\,\boldsymbol{\Gamma}_t\,|J\rangle\ ,\ |\mathcal{X}_I\rangle = \sum_J\left(S^{-1}\right)_{JI}|J\rangle\ ,\ S_{IJ} = \langle I|J\rangle\ .$$

RG evolution of the soft evolution operator

$$\mathbf{V}(b,M;y_{34}) = \bar{P}_q \mathrm{exp}\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\boldsymbol{\Gamma}_t(\alpha_S(q^2);y34)\right\}.$$

V fulfils the following evolution equation

$$\frac{d\mathbf{V}(b,Q;y_{34})}{d\ln\left(b_0^2/b^2\right)} = \boldsymbol{\Gamma}_t(\alpha_S(b_0^2/b^2);y_{34})\mathbf{V}(b,Q;y_{34})\,.$$

The solution to this equation can be written as

$$\mathbf{V}(b,Q) = \mathbf{K}(\alpha_S(b_0^2/b^2))\mathbf{V}^{(\mathrm{LO})}(\alpha_S(b_0^2/b^2),\alpha_S(Q^2))\mathbf{K}^{-1}(\alpha_S(Q^2))\,,$$

$$\mathbf{K}(\alpha_S) = \mathbf{1} + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n\mathbf{K}^{(n)}\,.$$

$$\frac{d\mathbf{V}^{(\mathrm{LO})}(\alpha_S,\alpha'_S)}{d\ln\alpha_S} = -\frac{1}{\beta_0}\boldsymbol{\Gamma}_t^{(1)}\mathbf{V}^{(\mathrm{LO})}(\alpha_S,\alpha'_S)\,,$$

$$\mathbf{V}^{(\mathrm{LO})}(\alpha_S(b_0^2/b^2),\alpha_S(Q^2)) = \exp\left\{\frac{1}{\beta_0}\boldsymbol{\Gamma}_t^{(1)}\ln\left(\frac{\alpha_S(Q^2)}{\alpha_S(b_0^2/b^2)}\right)\right\}.$$

$$V_{cc'}^{(\mathrm{LO})} = \frac{1}{\langle c|c\rangle}\langle c|\mathbf{V}^{(\mathrm{LO})}|c'\rangle = \sum_I R_{cI}\exp\left\{\frac{\lambda_I^{(1)}}{\beta_0}\int_{b_0^2/b^2}^{Q^2}\frac{dq^2}{q^2}\beta(\alpha_S(q^2))\right\}R_{Ic'}^{-1}\,,$$

$\lambda_I^{(1)}$ are the eigenvalues of first-order soft anomalous dimension matrix.

The NLO+NLL structure of the final-state radiation

The $\boldsymbol{H\Delta}$ factor can be organized as follows to all orders

$$\boldsymbol{H\Delta} \sim \sum_{\{C\}} \mathcal{H}^{\{C\}}(\alpha_S(\mu_R^2))\,\exp\{\mathcal{G}_{\{C\}}(\alpha_S(\mu_R^2)\}\,,$$

$$\mathcal{H}^{\{C\}}(\alpha_S) = \alpha_S^2 \left(\mathcal{H}^{\{C\},(0)} + \frac{\alpha_S}{\pi}\mathcal{H}^{\{C\},(1)} + \mathcal{O}(\alpha_S(\mu_R^2))\right)\ ,$$

$$\mathcal{G}_{\{C\}}(\alpha_S) = \underset{\text{NLL}\nearrow}{g^{(2)}_{\{C\}}}(\alpha_S) + \frac{\alpha_S}{\pi}\underset{\text{NNLL}\nearrow}{g^{(3)}_{\{C\}}}(\alpha_S) + \cdots$$

- At NLO+NLL ($Q = M$)

$$g_C^{(2)}(\alpha_S L) = \frac{\lambda_I^{(1)*} + \lambda_J^{(1)}}{\beta_0}\,\ln(1-\lambda)\,,$$

with $\lambda = \frac{1}{\pi}\beta_0\alpha_S(\mu_R^2)L$, $L = \ln\frac{Q^2b^2}{b_0^2}$.

$$\boldsymbol{H\Delta} \sim \sum_{\{C\}} \alpha_S^2 \left(\mathcal{H}^{\{C\},(0)}\exp\{g_C^{(2)}\} + \frac{\alpha_S}{\pi}\mathcal{H}^{\{C\},(1)}\right)\ ,$$

$$\mathcal{H}^{\{C\},(0)} = \langle\widetilde{\mathcal{M}}^{(0)}|\mathcal{X}_I\rangle\, S_{IJ}\,\langle\mathcal{X}_J|\widetilde{\mathcal{M}}^{(0)}\rangle$$

$$\mathcal{H}^{\{C\},(1)} = \left(\langle\widetilde{\mathcal{M}}^{(1)}|\mathcal{X}_I\rangle\,\langle\mathcal{X}_J|\widetilde{\mathcal{M}}^{(0)}\rangle + \langle\widetilde{\mathcal{M}}^{(0)}|\mathcal{X}_I\rangle\,\langle\mathcal{X}_J|\widetilde{\mathcal{M}}^{(1)}\rangle\right) S_{IJ}$$

Resummation in MC event generators

q_T-resummation is effectively performed by Monte Carlo event generators.

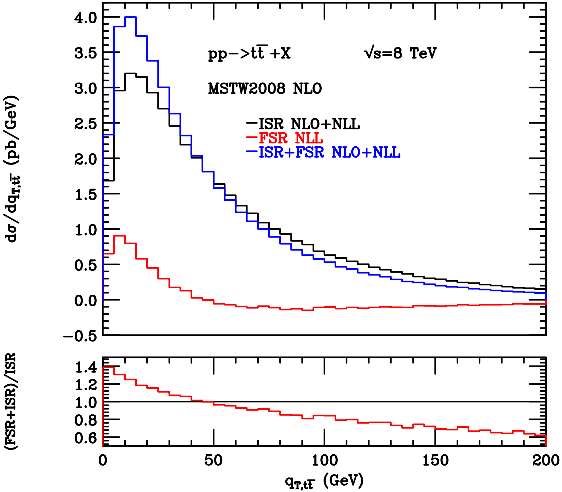
- Based on the QCD factorization at the level of squared matrix elements.

In the case of strongly interacting final state particles the additional soft singularities are colour correlated.

- The large logs due to the soft radiation off the final state and due to the initial-final state interference are not under control through the shower.

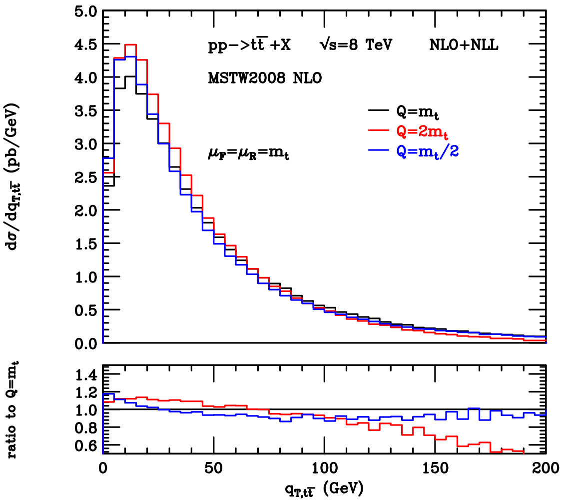
Preliminary results at NLO+NLL

- Large distortion of the spectrum due to soft radiation off the top quarks.



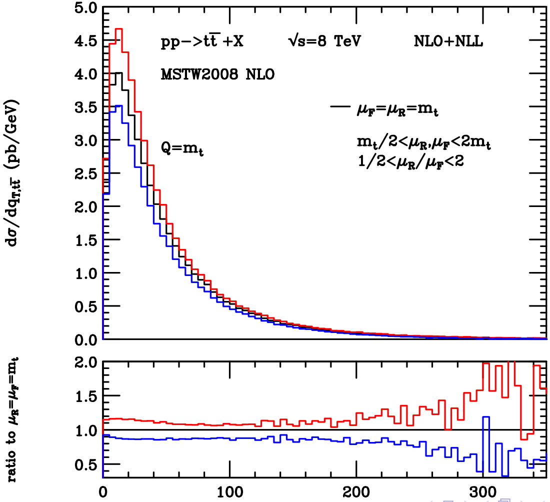
Resummation scale variation

- The resummation scale variation is of the order of 15% up to $q_T = 100\,\mathrm{GeV}$. Increases at large transverse momenta.



Renormalisation and factorisation scale variation

- The renormalisation and factorisation scale variation is of the order of 15% in the low-q_T region.
- shrinks a bit in the intermediate region, and increases at large transverse momenta, reaching to 100%.



Comparison to the data

- Both CMS and ATLAS experiments measured the q_T distribution of the $t\bar{t}$ pair at the LHC at $\sqrt{s} = 7\,\mathrm{TeV}$. CMS-PAS-TOP-11-013, G. Aad et al. [ATLAS Collaboration], 2013.
- Very recently ATLAS has also published the measurements of normalized differential cross sections at $\sqrt{s} = 8\,\mathrm{TeV}$ M. Aaboud et al. [ATLAS Collaboration], 2016.

$p_{T,t\bar{t}}$ [GeV]	$\frac{1}{\sigma}\frac{d\sigma}{dp_{T,t\bar{t}}}$ [TeV^{-1}] ATLAS	$\frac{1}{\sigma}\frac{d\sigma}{dp_{T,t\bar{t}}}$ [TeV^{-1}] NLO+NLL
0-30	14.3 ± 1.0	14.96 ± 0.99
30-70	7.60 ± 0.16	7.81 ± 0.36
70-120	2.94 ± 0.28	2.84 ± 0.28
120-180	1.14 ± 0.12	0.99 ± 0.08
180-250	0.42 ± 0.04	0.34 ± 0.02
250-350	0.143 ± 0.018	0.096 ± 0.020
350-1000	0.0099 ± 0.0015	0.0062 ± 0.0040

Table : Normalized $p_{T,t\bar{t}}$ distribution at $\sqrt{s} = 8\,\mathrm{TeV}$.

- Good agreement with the experimental results along the whole range of transverse momenta.
- The theoretical uncertainties are of the order of experimental uncertainties.

q_T-subtraction

Knowledge of the low q_T limit is essential also for the fixed order calculation in the q_T-subtraction formalism. It has been originally proposed for the production of colourless high-mass systems in hadron collisions. [Catani, Grazzini (2007)].

- $pp \to H$ [Catani, Grazzini (2007)].
- $pp \to V$ [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)].
- $pp \to \gamma\gamma$ [Catani, Cieri, Ferrera, de Florian, Grazzini (2011)].
- $pp \to WH$ [Ferrera, Grazzini, Tramontano (2011)].
- $pp \to Z\gamma$ [Grazzini, Kallweit, Rathlev, Torre (2013)].
- $pp \to ZZ$ [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)].
- $pp \to W^+W^-$ [Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)].
- $pp \to ZH$ [Ferrera, Grazzini, Tramontano (2014)].
- $pp \to WZ$ [Grazzini, Kallweit, Rathlev, Wiesemann (2016)].

  (talk by M. Wiesemann)
- $pp \to HH$ [de Florian, Grazzini, Hanga, Kallweit, Lindert, Meierhöfer, Mazzitelli, Rathlev (2016)]. (talk by J. Mazzitelli)

q_T-subtraction for $t\bar{t}$

- The fully differential cross section at N(NLO):

$$d\sigma^{t\bar{t}}_{\mathrm{N(NLO)}} = \mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}} \otimes d\sigma^{t\bar{t}}_{\mathrm{LO}} + \left[d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}} - d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}} \right] .$$

Regular as $q_T \to 0$

- $\mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}}$ is the hard factor, which contains information on the virtual corrections to the LO process.
- $d\sigma^{t\bar{t}}_{\mathrm{LO}}$ is the Born cross section.
- $d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}}$ is the N(LO) cross section of $t\bar{t}$+jet(s) process.
- $d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}}$ is the counterterm, which can be derived by expanding the resummation formula.

Azimuthal correlations

- Production of a colourless system
  - The gluonic collinear functions are the only source of azimuthal correlations

$$C^{\mu\nu}_{ga}(z;p_1,p_2,\mathbf{b}) = d^{\mu,\nu}(p_1,p_2)C_{ga}(z)+D^{\mu,\nu}(p_1,p_2;\mathbf{b})G_{ga}(z)$$

- Top-quark pair production

$$\boldsymbol{\Delta}(b,M;y_{34},\phi_3) = \mathbf{V}^{\dagger}(b,M;y_{34})\mathbf{D}\left(\alpha_S;\phi_{3b},y_{34}\right)\mathbf{V}(b,M;y_{34})\,.$$

  - Additional azimuthal correlations produced by the dynamics of soft-parton radiation, embodied in **D**.

$$\mathbf{D}\left(\alpha_S;\phi_{3b},y_{34}\right) = 1+\frac{\alpha_S}{\pi}\mathbf{D}^{(1)}\left(\phi_{3b},y_{34}\right)+\mathcal{O}(\alpha_S^2)$$

$\langle\mathbf{D}\left(\alpha_S\left(b_0^2/b^2\right);\phi_{3b},y_{34}\right)\rangle_{\text{av.}} = 1 \longrightarrow$ vanishing contribution to $\langle\sigma\rangle_{\text{av.}}$ at $\mathcal{O}(\alpha_S)$

But contributes at $\mathcal{O}(\alpha_S^2)$ due to the interference of the initial-final state azimuthal correlations

$\longrightarrow$ non-trivial integration over the azimuthal angle (computed analytically!)

Our fixed-order implementation

Up to NLO our implementation is based on

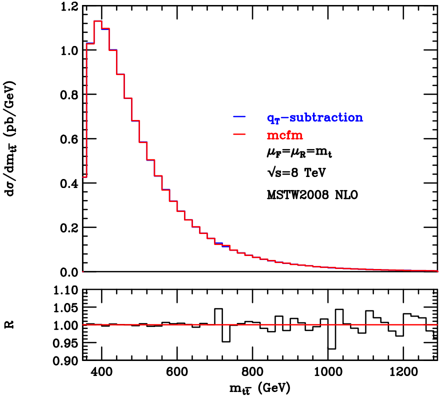
- The scattering amplitudes and phase space generation of MCFM program.
- We use the corresponding routines of Higgs boson production code HNNLO and vector boson production code DYNNLO, suitably modified for the $t\bar{t}$ production.

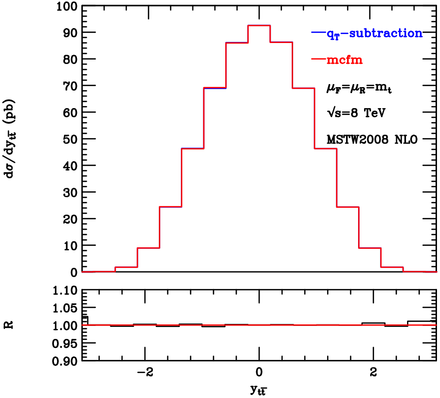
At NNLO accuracy the $t\bar{t}$ + jet cross section is evaluated by using the MUNICH code which provides:

- Fully automatic implementation of the NLO dipole subtraction formalism.
- Interface to the OPENLOOPS one-loop generator.

Results at NLO

- Distributions for the $t\bar{t}$ system.

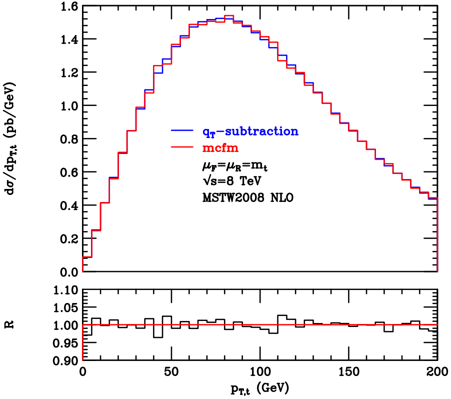


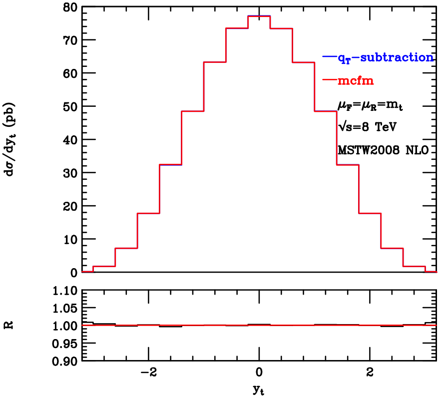


- Very good agreement!

Results at NLO

- Distributions for the top quark.





- Very good agreement!

Results at NNLO

Cross section [pb]	$\mathcal{O}(\alpha_S^4)_{qg}$	$\mathcal{O}(\alpha_S^4)_{q(\bar{q})q'}$
q_T subtraction	-2.25(5)	0.151(3)
Top++	-2.253	0.148

Table : $\mathcal{O}(\alpha_S^4)$ contribution to the total cross section for $t\bar{t}$ production at the LHC at $\sqrt{s} = 8\,\mathrm{TeV}$.

Cross section [pb]	$\mathcal{O}(\alpha_S^4)_{qg}$	$\mathcal{O}(\alpha_S^4)_{q(\bar{q})q'}$
q_T subtraction	-61.5(5)	1.33(1)
Top++	-61.53	1.33

Table : $\mathcal{O}(\alpha_S^4)$ contribution to the total cross section for $t\bar{t}$ production at the LHC at $\sqrt{s} = 2\,\mathrm{TeV}$.

$$qg = qg + \bar{q}g, \quad q(\bar{q})q' = qq + \bar{q}\bar{q} + qq' + \bar{q}\bar{q}' + q\bar{q}' + \bar{q}q'$$

Summary

- I have presented the computation of the q_T-resummed cross section for the $t\bar{t}$ production at hadron colliders at NLO+NLL in QCD.
- The calculation is more complicated with respect to hadroproduction of colourless systems due to the additional radiation of soft gluons off the top quarks.
- The resummation of large logs due to the soft radiation off top quarks leads to a sizeable distortion of the transverse momentum spectrum.
- Within the uncertainties our results agree with the most up-to-date ATLAS measurement of the q_T distribution.
- The uncertainties due to the scale variations are large, being of the order of experimental uncertainties.
- We have used the knowledge of the low q_T behaviour of the amplitudes to extend the q_T subtraction method for the $t\bar{t}$ production at hadron colliders at NLO and NNLO in all non-diagonal channels.
- We have implemented the calculation in a fully-differential Monte Carlo program and found good agreement with the known results.

Backup Slides

Comparision to the fixed order

- The spectrum for $Q = \frac{m_t}{2}$ matches very well the fixed order curve at large transverse momenta.
- For the scales $Q = m_t$ and $Q = 2m_t$ the need of a switching procedure at intermediate values is evident.