One loop further: 4 and 5 loop physics

Ben Ruijl*, Franz Herzog, Takahiro Ueda, Jos Vermaseren, Andreas Vogt

*LCDS Leiden and Nikhef Amsterdam

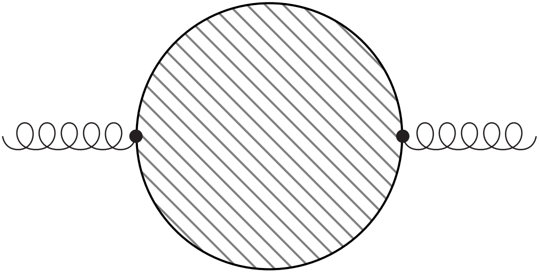
Sep 07, 2016

Universiteit Leiden
Leiden Centre of Data Science

Nikhef

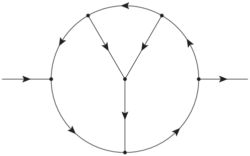
Forcer package

- We have built a program that calculates four-loop massless propagator diagrams
- Express diagrams as linear combinations of diagrams with fewer propagators



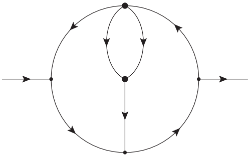
IBP identities

Through integration by parts (IBP) identities we find rules to remove lines:



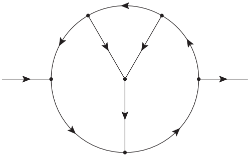
IBP identities

Through integration by parts (IBP) identities we find rules to remove lines:



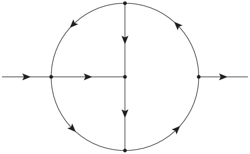
IBP identities

Through integration by parts (IBP) identities we find rules to remove lines:



IBP identities

Through integration by parts (IBP) identities we find rules to remove lines:



Identify substructures

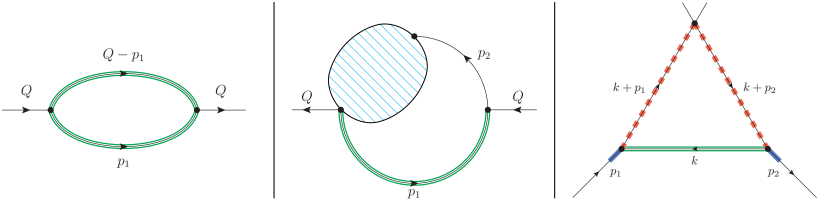


Figure: Guaranteed to remove a green or blue line

Triangle generalizations: diamonds [Ruijl,Ueda,Vermaseren '15]

Example reduction

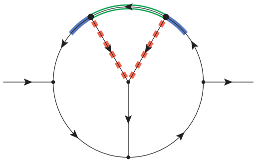


Figure: Reduction of the Benz diagram

Example reduction

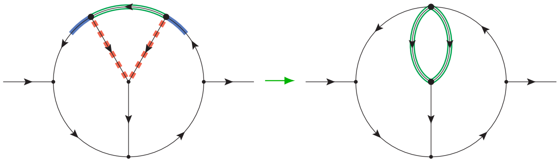


Figure: Reduction of the Benz diagram

Example reduction

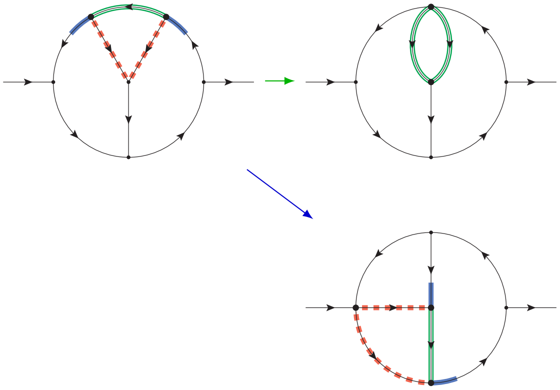


Figure: Reduction of the Benz diagram

Example reduction

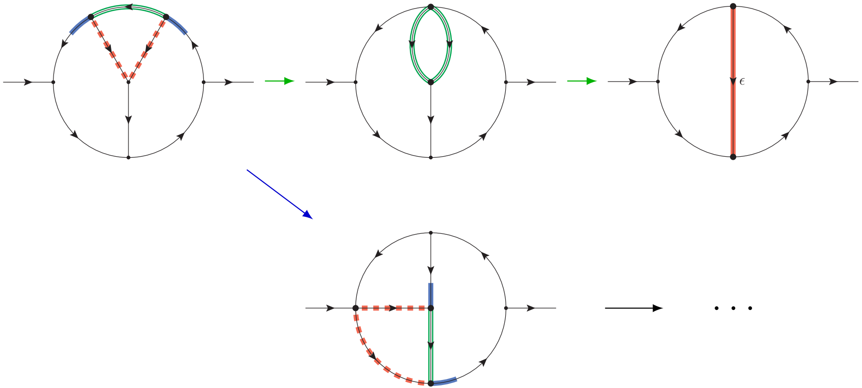


Figure: Reduction of the Benz diagram

3-loop reduction graph

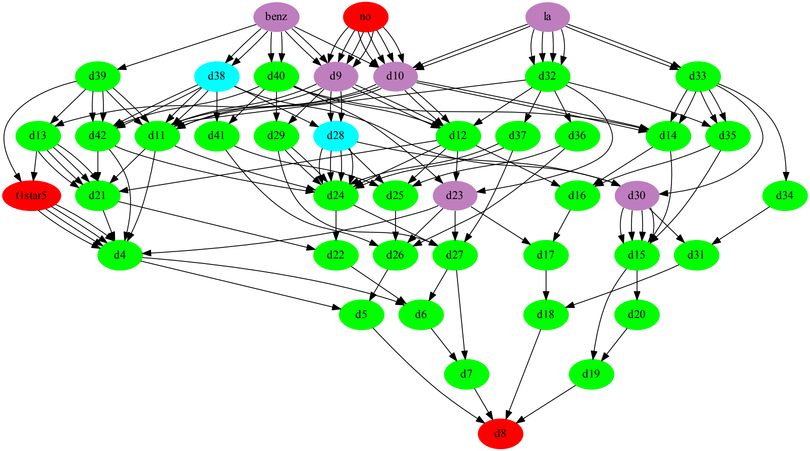


Figure: Reduction graph for 3 loop diagrams

4-loop reduction graph

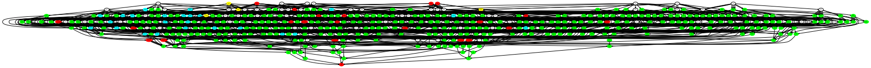


Figure: Reduction graph for 4 loop diagrams

4-loop reduction graph

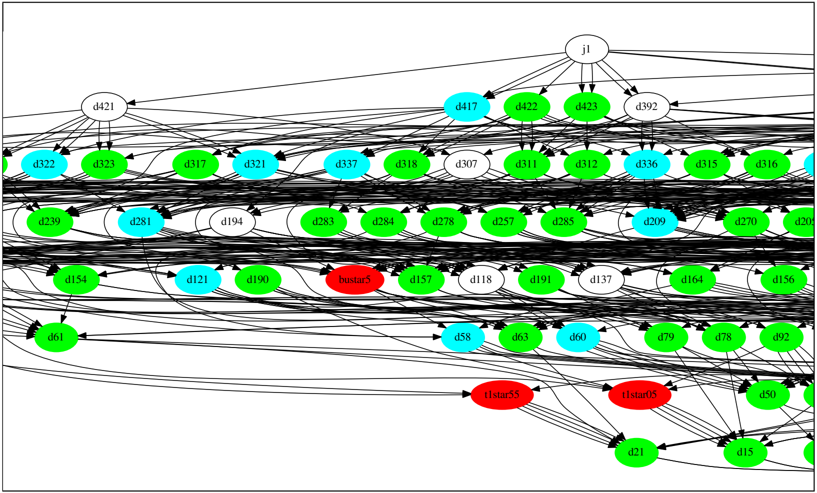
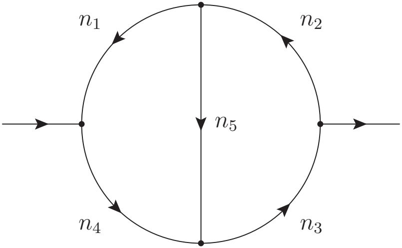


Figure: Part of reduction graph for 4 loops

Manual solutions

- 21 topologies do not have substructures
- Solve recursion relations parametrically
- Much faster than Laporta methods



$$I(n_1, n_2, n_3, n_4, n_5) = n_1 I(n_1 - 1, n_2, n_3, n_4, n_5) + n_2 I(n_1, n_2 + 1, n_3, n_4, n_5) + \dots$$

$$I(n_1, n_2, n_3, n_4, n_5) = n_3 I(n_1, n_2, n_3 - 1, n_4, n_5) + n_3 I(n_1, n_2 + 1, n_3, n_4, n_5) + \dots$$

Parametric solution vs. Laporta (I)

$$
\begin{aligned}
1)\ I(n_1,n_2,n_3,n_4,n_5) &= +\,I(n_1-1,n_2,n_3,n_4,n_5)\\
&\quad +\,n_2 I(n_1,n_2+1,n_3,n_4,n_5)\\
2)\ I(n_1,n_2,n_3,n_4,n_5) &= +\,I(n_1,n_2,n_3,n_4-1,n_5)\\
&\quad +\,I(n_1,n_2+1,n_3,n_4,n_5)\\
\Rightarrow I(n_1,n_2,n_3,n_4,n_5) &= \frac{1}{n_2-1}\Big(n_2 I(n_1,n_2,n_3,n_4-1,n_5)\\
&\qquad - I(n_1-1,n_2,n_3,n_4,n_5)\Big)
\end{aligned}
$$

Next step in reduction

Solve $I(n_1, 1, n_3, n_4, n_5)$ since $n_2 > 2$ will have $n_1 = 0$ or $n_4 = 0$!

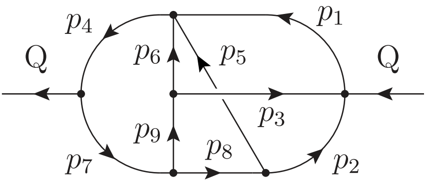
Parametric solution vs. Laporta (II)

Laporta for $n_2 = 2$:

$$1)\ I(1,2,1,1,1) = I(0,2,1,1,1) + 2I(1,3,1,1,1)$$
$$2)\ I(1,2,1,1,1) = I(1,2,1,0,1) + I(1,3,1,1,1)$$
$$\Rightarrow I(1,2,1,1,1) = 2I(1,2,1,0,1) - I(0,2,1,1,1)$$

- System needs to be brute forced again for every $n_2 > 2$
- A lot of equations are generated that drop out of the answer

Public enemy #1



- 9 propagators that should go to 1
- 5 irreducible dot products that should go to 0
- Took 3 months to find solution!

Reduction rule

```
id,ifmatch->bubu1,
    Z(n1?pos_,n2?pos_,n3?pos_,n4?pos_,n5?pos_,n6?pos_,n7?pos_,
    n8?pos_,n9?pos_,n10?neg0_,n11?neg0_,n12?neg0_,n13?neg0_,n14?neg_)
       = -rat(1,-2*ep-2*n1-n3-n6-n12-n14+4)*(
+Z(-1+n1,-1+n2,n3,n4,1+n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(-n5,1)
+Z(-1+n1,1+n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(n2,1)
+Z(-1+n1,1+n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,1+n14)*rat(-n2,1)
+Z(-1+n1,n2,1+n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,n12,n13,1+n14)*rat(-n3,1)
+Z(-1+n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,1+n12,n13,1+n14)*rat(-n12,1)
+Z(-1+n1,n2,n3,n4,1+n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,1+n14)*rat(n5,1)
+Z(-1+n1,n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,n11,n12,1+n13,1+n14)*rat(-n13,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,1+n12,n13,1+n14)*rat(2*n12,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,2+n14)*rat(2*n14+2,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,n8,n9,1+n10,-1+n11,n12,n13,1+n14)*rat(-n10,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,1+n12,n13,n14)*rat(-n12,1)
+Z(-1+n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(-n2+n5,1)
+Z(n1,-1+n2,-1+n3,n4,n5,n6,n7,n8,n9,n10,n11,1+n12,n13,1+n14)*rat(n12,1)
+Z(n1,-1+n2,-1+n3,n4,n5,n6,n7,n8,n9,n10,n11,n12,n13,2+n14)*rat(1+n14,1)
+Z(n1,-1+n2,1+n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,n12,n13,1+n14)*rat(n3,1)
```

```
+Z(n1,-1+n2,n3,-1+n4,n5,n6,n7,n8,n9,n10,1+n11,n12,n13,1+n14)*rat(n11,1)
+Z(n1,-1+n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,1+n12,n13,1+n14)*rat(n12,1)
+Z(n1,-1+n2,n3,n4,n5,-1+n6,n7,n8,n9,n10,n11,n12,n13,2+n14)*rat(-n14-1,1)
+Z(n1,-1+n2,n3,n4,n5,n6,-1+n7,1+n8,n9,n10,n11,n12,n13,1+n14)*rat(2*n8,1)
+Z(n1,-1+n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,1+n11,n12,n13,1+n14)*rat(-n11,1)
+Z(n1,-1+n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,n11,n12,1+n13,1+n14)*rat(-n13,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,1+n12,n13,1+n14)*rat(-2*n12,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,2+n14)*rat(-2*n14-2,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,1+n8,-1+n9,n10,n11,n12,n13,1+n14)*rat(-n8,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,1+n12,n13,1+n14)*rat(-2*n12,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,n8,n9,1+n10,-1+n11,n12,n13,1+n14)*rat(n10,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,n8,n9,1+n10,n11,n12,n13,1+n14)*rat(-2*n10,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,n8,n9,n10,1+n11,n12,n13,1+n14)*rat(-n11,1)
+Z(n1,-1+n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,1+n12,n13,n14)*rat(-n12,1)
+Z(n1,1+n2,n3,n4,-1+n5,n6,n7,n8,n9,-1+n10,n11,n12,n13,1+n14)*rat(-n2,1)
+Z(n1,1+n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(-n2,1)
+Z(n1,1+n2,n3,n4,n5,n6,n7,-1+n8,n9,-1+n10,n11,n12,n13,1+n14)*rat(2*n2,1)
+Z(n1,1+n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,1+n14)*rat(n2,1)
+Z(n1,n2,-1+n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,1+n12,n13,1+n14)*rat(-n12,1)
+Z(n1,n2,-1+n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,2+n14)*rat(-n14-1,1)
+Z(n1,n2,1+n3,n4,n5,n6,n7,n8,-1+n9,-1+n10,n11,n12,n13,1+n14)*rat(n3,1)
```

```
+Z(n1,n2,n3,n4,-1+n5,n6,-1+n7,n8,n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,-1+n9,n10,n11,n12,1+n13,1+n14)*rat(-n13,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,-1+n9,n10,n11,n12,n13,2+n14)*rat(1+n14,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,-1+n10,n11,1+n12,n13,1+n14)*rat(n12,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,-1+n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,1+n12,n13,1+n14)*rat(n12,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,n2,n3,n4,-1+n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(-n2+n8-n13,1)
+Z(n1,n2,n3,n4,n5,-1+n6,n7,-1+n8,n9,n10,n11,n12,n13,2+n14)*rat(1+n14,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,-1+n8,n9,n10,1+n11,n12,n13,1+n14)*rat(n11,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,-1+n8,n9,n10,n11,n12,1+n13,1+n14)*rat(-2*n13,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,n8,n9,-1+n10,1+n11,n12,n13,1+n14)*rat(n11,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,n11,-1+n12,n13,2+n14)*rat(1+n14,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,n2,n3,n4,n5,n6,-1+n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(-2*ep-2*n4-1,1)
+Z(n1,n2,n3,n4,n5,n6,1+n7,-1+n8,n9,n10,n11,n12,n13,1+n14)*rat(-n7,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,-1+n9,n10,n11,n12,1+n13,1+n14)*rat(n13,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,-1+n9,n10,n11,n12,n13,2+n14)*rat(-2*n14-2,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,-1+n10,1+n11,n12,n13,1+n14)*rat(-n11,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,-1+n10,n11,1+n12,n13,1+n14)*rat(-2*n12,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,-1+n10,n11,n12,n13,2+n14)*rat(-2*n14-2,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,1+n10,n11,n12,n13,1+n14)*rat(2*n10,1)
```

```
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,-1+n11,n12,1+n13,1+n14)*rat(-2*n13,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,1+n11,n12,n13,1+n14)*rat(n11,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,-1+n12,n13,2+n14)*rat(-n14-1,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,1+n12,n13,1+n14)*rat(-2*n12,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,1+n13,1+n14)*rat(-3*n13,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,1+n14)*rat(10*ep+2*n1,1)
+Z(n1,n2,n3,n4,n5,n6,n7,-1+n8,n9,n10,n11,n12,n13,2+n14)*rat(-2*n14-2,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,-1+n10,n11,1+n12,n13,1+n14)*rat(-n12,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,-1+n10,n11,n12,1+n13,1+n14)*rat(-n13,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,n10,-1+n11,n12,n13,2+n14)*rat(-n14-1,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,n12,1+n13,1+n14)*rat(-n13,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,-1+n9,n10,n11,n12,n13,1+n14)*rat(-4*ep-2*n1-n3,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,-1+n10,n11,n12,n13,1+n14)*rat(-n5+1,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,1+n10,-1+n11,n12,n13,1+n14)*rat(n10,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,-1+n11,n12,n13,1+n14)*rat(2*ep+n5+2*n8
    +n9+n10+n11-5,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,1+n12,n13,n14)*rat(n12,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,n12,-1+n13,1+n14)*rat(-2*ep-n5-2*n8
    -n9-n11-n14+3,1)
+Z(n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11,n12,n13,1+n14)*rat(2*ep+n2+n7+
    2*n8+n9+n11+n14-4,1)
);
```

First results and benchmarks

- Reproduced 4-loop QCD β-function [Ritbergen, Vermaseren, Larin '97; Czakon '04]
- All ξ and ϵ 4-loop solution for QCD propagators and vertices [new!]

β_3 no gauge	10 minutes
β_3 1 gauge	38 minutes
β_3 all gauge	8.5 hours
no1(2,2,2,2,2,2,2,2,2,2,2,-1,-1,-1)	42 minutes

Table: Benchmark on 24 core machine

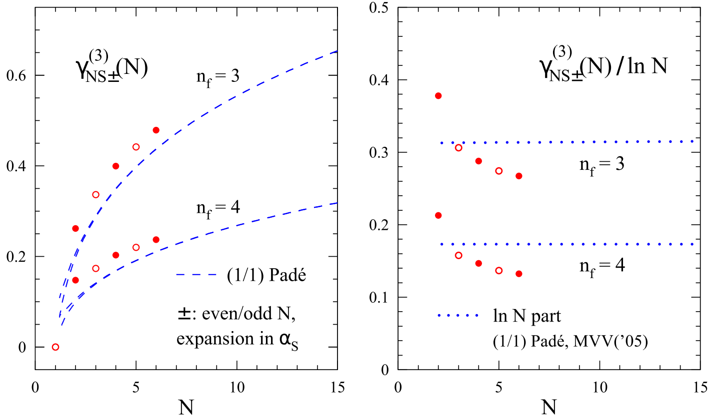
4-loop splitting functions

- Can we calculate the 4-loop corrections to the PDF evolution?
- Calculate Mellin moments N for as many N as possible
- Each $N \to N+2$ increases weight by 4
- 4-loop time: more than 1000 times 3-loop time

$$\gamma_{ij} = -P_{ij} = \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n P_{ij}^{(n-1)} \quad i,j = q,g$$

Non-singlet (NS): combinations of γ_{qq}

N3LO corrections to γ_{NS}



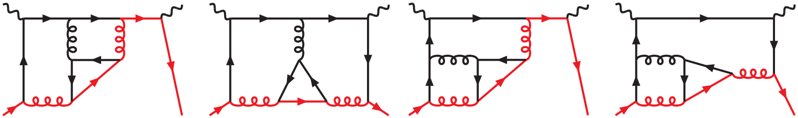
- $N = 2, 3, 4$ in agreement with [Baikov,Chetyrkin '06, Velizhanin '13,'14]
- Not enough moments yet to improve Padé of $\gamma^{(3)}_{\text{cusp}}$

Example result

$$
\begin{aligned}
\gamma^{(3)}_{\rm gg}(N=4) \;=\; & C_A^4 \left(\frac{1502628149}{3375000} + \frac{1146397}{11250}\zeta_3 - \frac{504}{5}\zeta_5 \right) + \frac{d_A^{abcd}\, d_A^{abcd}}{n_a} \left(\frac{21623}{150} \right. \\
& \left. + \frac{15596}{15}\zeta_3 - \frac{6048}{5}\zeta_5 \right) - n_f C_A^3 \left(\frac{20580892841}{72900000} + \frac{12550223}{22500}\zeta_3 - \frac{8613}{25}\zeta_4 - \frac{4316}{27}\zeta_5 \right) \\
& + n_f \frac{d_R^{abcd}\, d_A^{abcd}}{n_a} \left(\frac{160091}{675} + \frac{80072}{225}\zeta_3 - \frac{48016}{45}\zeta_5 \right) - n_f C_A^2 C_F \left(\frac{4212122951}{41006250} \right. \\
& \left. - \frac{1170784}{5625}\zeta_3 + \frac{418198}{1125}\zeta_4 - \frac{17636}{45}\zeta_5 \right) + n_f C_A C_F^2 \left(\frac{1913110089023}{26244000000} + \frac{39313783}{101250}\zeta_3 \right. \\
& \left. + \frac{26741}{750}\zeta_4 - \frac{3082}{5}\zeta_5 \right) + n_f C_F^3 \left(\frac{34764568601}{2099520000} - \frac{958343}{40500}\zeta_3 - \frac{18997}{2250}\zeta_4 + \frac{908}{45}\zeta_5 \right) \\
& - n_f^2 C_A^2 \left(\frac{3250393649}{218700000} - \frac{2969291}{20250}\zeta_3 + \frac{1566}{25}\zeta_4 + \frac{1276}{135}\zeta_5 \right) - n_f^2 C_F^2 \left(\frac{275622924731}{26244000000} \right. \\
& \left. - \frac{253369}{10125}\zeta_3 + \frac{1078}{225}\zeta_4 \right) + n_f^2 C_A C_F \left(\frac{136020246173}{3280500000} - \frac{1672751}{10125}\zeta_3 + \frac{15172}{225}\zeta_4 \right) \\
& + n_f^2 \frac{d_R^{abcd}\, d_R^{abcd}}{n_a} \left(\frac{75788}{675} + \frac{3008}{15}\zeta_3 - \frac{20416}{45}\zeta_5 \right) + n_f^3 C_F \left(\frac{1780699}{24300000} - \frac{484}{675}\zeta_3 \right) \\
& - n_f^3 C_A \left(\frac{20440457}{21870000} - \frac{1888}{405}\zeta_3 \right)
\end{aligned}
$$

All-N results for n_f^3 and n_f^2

All 3-loop graphs (with insertion on gluon line) for n_f^2 in γ_{qq}:



- Simple topologies for forcer $\rightarrow$ high moments
- Analytic form in N reconstructed with LLL-method
  [Velizhanin '12; Moch,Vermaseren,Vogt '14]
- Recent success: n_f^3 for γ_{qg} required $N = 44$
- Singlet splitting function matrix is now known at leading n_f
  [new; Davies,RUVV]

Quark cusp anomalous dimension

Large n_c contribution example:

$$\gamma_{\rm ns}^{(3)}(N)|_{C_F n_c n_f^2} = \frac{127}{18} + \frac{1}{81}\Big(\frac{20681}{2}\,\eta + 2119\,S_1 - 2275\,\eta^2 - 20460\,D_1^2 + 3392\,S_1\eta - 5036\,S_2\Big) + \frac{4}{81}\Big(118\,\eta^3 - 886\,D_1^3 - 914\,S_1\eta^2 - 848\,S_1 D_1^2 - 152\,S_{1,2} - 416\,S_2\eta - 152\,S_{2,1} + 1148\,S_3\Big) + \frac{8}{27}\Big(-57\,D_1^4 + 18\,S_1\eta^3 - 24\,S_1 D_1^3 + 2\,S_2\eta^2 + 128\,S_2 D_1^2 - 8\,S_3\eta + 40\,S_{1,3} + 80\,S_{2,2} + 120\,S_{3,1} - 159\,S_4\Big) + \frac{8}{9}\Big(-6\,\eta^5 - 12\,D_1^5 + 10\,S_1\eta^4 - 24\,S_1 D_1^4 + 8\,S_2\eta^3 + 4\,S_3\eta^2 - 8\,S_3 D_1^2 + 4\,S_{3,1}\eta - 8\,S_{1,3,1} + 4\,S_{1,4} - 8\,S_{2,3} - 16\,S_{3,2} - 2\,S_4\eta - 20\,S_{4,1} + 24\,S_5\Big) + \text{much simpler } \zeta_3, \zeta_4 \text{ terms}$$

From ln N coefficient in large-N limit:

$$\gamma_{\rm cusp}^{(3)} = \ldots + C_F C_A n_f^2\left(\frac{923}{81} - \frac{608}{81}\zeta_2 + \frac{2240}{27}\zeta_3 - \frac{112}{3}\zeta_4\right) + C_F^2 n_f^2\left(\frac{2392}{81} - \frac{640}{9}\zeta_3 + 32\,\zeta_4\right) - C_F n_f^3\left(\frac{32}{81} - \frac{64}{27}\zeta_3\right)$$

Five loops

- Can we go to five loops?
- Recompute 5-loop QCD β-function [Baikov, Chetyrkin, Kühn '16] with general colour group and gauge? [see also: Luthe,Maier,Marquard,Schröder '16]
- 5-loop Forcer is hard:
  - Unknown master integrals
  - More than 200 manual reductions!

Five loops

- We are interested in anomalous dimensions
- Thus, we only need the UV pole part
- Can we IR rearrange to create a *carpet* diagram?

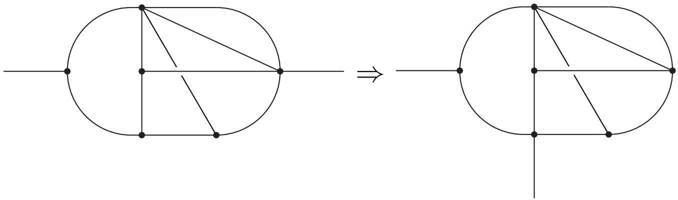
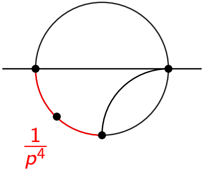


Figure: The graphs have the same UV poles

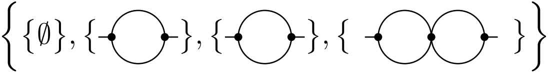
Five loops

- In general, this creates IR divergences
- Subtract UV and IR all using R^* [Chetyrkin,Smirnov,Larin,. . .]
- This involves carefully identifying and subtracting all counter-terms
- R^* has its own problems: commutativity issues, high-rank tensor reductions, . . .

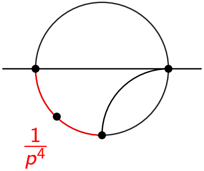
Subdivergences



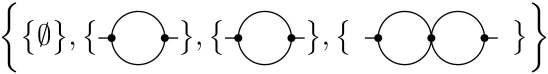
UV subdiagrams:



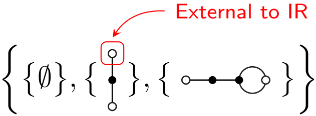
Subdivergences



UV subdiagrams:

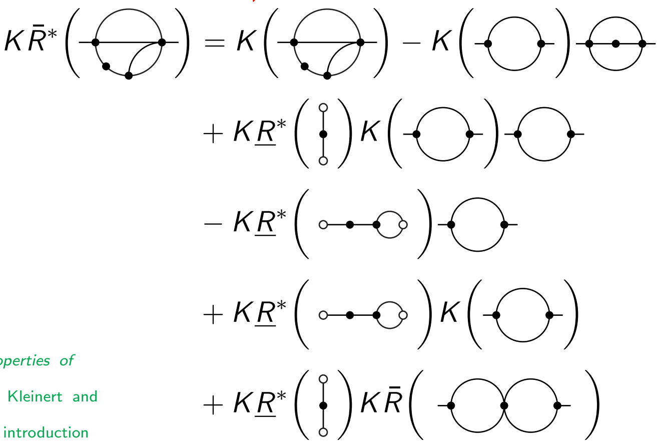


IR subdiagrams:



Superficial UV divergence

Pole part: $K(\frac{1}{\epsilon} + 1 + \epsilon) = \frac{1}{\epsilon}$



see *Critical Properties of ϕ^4-Theories* by Kleinert and Schulte for an introduction

Conclusion

- Very fast four-loop reductions beyond the scope of Laporta methods
- Calculated Mellin moments, including all-N generalizations
- Extensions to five loop is difficult, but possible
- R^* of interest for 4-loop high N or 5-loop splitting functions

Acknowledgements

This work is supported by the ERC Advanced Grant no. 320651, "HEPGAME"